ONTARIO
PUBLIC SCHOOL
ARITHMETIC

Price 10 Cents

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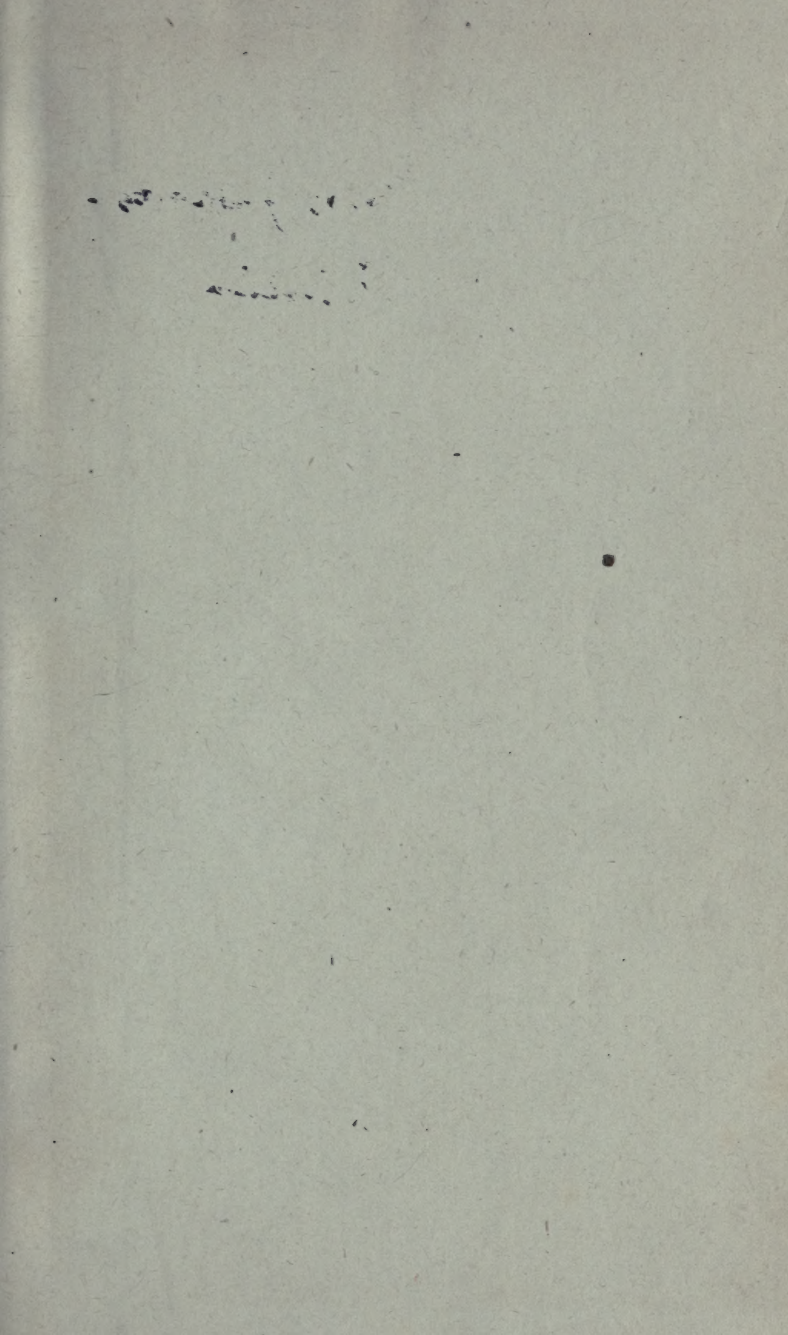

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THE PUBLIC SCHOOL ARITHMETIC

SECTION I

MEASUREMENT AND NUMBER

IN EACH of such terms as: 5 inches, 3 quarts, 7 pounds, $4, 8 hours, three things are named. For example, in 5 inches there is:

1. A **quantity** measured, or to be measured, namely, a five-inch length.
2. The **unit** used in measuring, namely, a one-inch length.
3. The **number** obtained by counting how many times the *unit* is contained in the quantity, namely, *five*, the symbol for which is 5.

Number comes from measuring some *quantity* by a unit—that is, from counting *how many* units the quantity contains.

The unit used depends upon the quantity to be measured. Sometimes it is one pound, sometimes one yard, one dollar, one day, one dozen, one ten, but it is always a part of the quantity measured.

EXERCISE 1

(Oral and Objective)

1. Name some quantities you have measured. What unit did you use in each case?

2. What unit is used to measure: a quantity of milk? of cheese? of potatoes? of eggs? of time?

3. Name some materials, quantities of which are measured by: the peck, the foot, the acre, the pair, the head, the twelve, the hundred, the thousand.

4. Measure a quantity of water, using a pint, a quart. What is the number obtained in each case? How many pints are there in a quart?

5. Measure a quantity of sand by the pint and by the gallon. What is the number obtained in each case? How many pints are there in a gallon? How many quarts?

6. Using the length of a pencil as unit, measure the length of the school desk, of the teacher's table, of the black-board. What is the number obtained in each case?

7. Measure a piece of paper three inches long. Use this as unit and find a quantity of six units. How many two-inch units would there be? How many one-inch units?

8. Name the number and the unit in each of these: 7 cents, 24 couples, 8 twos, 4 ten-dollar bills, 6 groups of five.

9. By measuring, find the number of ounces in one pound. How many pounds in 32 ounces? What unit is used in each case?

10. Measure a peck, using as unit a pint, a gallon, a quart.

11. Measure one yard, using as unit one inch, one foot.

12. Measure these two lines:

(a) ━━━━━━━━━━

(b) ━━━━━━━━━━━━━━━━━━━━━━━━

13. Measure the long line, using the short one as unit.

14. If the short line stands for three feet, for what will the long one stand?

15. Using the length of the short line as unit, draw a line the length of which would be 6; 4; 9.

16. The measurement of the length of a string is 4. Show how to find the length of the unit used.

17. Use objects and measure 12 of them by taking 4 of them as unit; 30 of them by taking 6 of them as unit; 8 by taking 2 as unit; 18 by taking 9 as unit; 40 by taking 10 as unit.

NOTATION AND NUMERATION

Numbers are expressed either in symbols or in words.

The expression of numbers by symbols is called **notation.**

The expression of numbers in words is called **numeration.**

The symbols generally used are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. These symbols are also called **numerals, digits,** or **figures.** Each of the first 9 digits has a value of its own, while 0, which is called zero, or naught, or the cipher, has no value, but is used merely to keep other digits in their proper places.

EXERCISE 2

(Review of Numbers from 1,000 to 999,999)

1. Write in figures to dictation:

Thirty-five, one hundred and thirty-five, seventy-eight, one hundred and seventy-eight, ninety-six, one hundred and ninety-six, eighty-three, one hundred and eighty-three, fifty-four, two hundred and fifty-four, sixty-five, two hundred and sixty-five, three hundred and forty-seven, four hundred and twenty-nine, five hundred and seventeen, seven hundred and eighty-one, eight hundred and eighty-eight, nine hundred and forty-three, nine hundred and seven.

2. Read the following numbers:

268, 350, 489, 605, 782, 948, 108, 600, 930, 755, 400, 801.

3. Place in order from the least to the greatest the following numbers:

    (a) 489, 627, 268, 307.
    (b) 608, 900, 57, 169, 475.
    (c) 768, 982, 750, 87, 408.
    (d) 490, 925, 363, 704, 682, 800, 78.

4. Write in order all the numbers from 523 to 541; from 597 to 629; from 857 to 888.

5. Write in figures:

One thousand three hundred and twenty-five

Two thousand six hundred and eighty-two

Four thousand eight hundred and seven

Six thousand and ten

Seven thousand nine hundred and seventy

Nine thousand one hundred and ninety-seven

Ten thousand.

6. Read the following numbers:

1000, 1111, 1462, 1589, 1400, 2682, 3908, 4005, 7815, 8694, 9217, 8402, 7000, 9001, 8024, 10,000.

7. Place in order from least to greatest the following numbers:

(a) 1468, 1372, 5468, 4271.

(b) 3927, 7000, 2601, 48, 793.

(c) 6429, 8003, 2427, 9682, 3890.

8. Write in figures:

Twenty-five thousand three hundred and seventy-five

Fifty-eight thousand six hundred and four

Ninety-two thousand eight hundred and twenty-one

One hundred thousand and one

Seven hundred thousand six hundred and fifty

Eight hundred thousand six hundred and fifty

Eight hundred and three thousand five hundred and sixty-two

Nine hundred and ninety-nine thousand nine hundred and ninety-nine.

9. Read the following numbers:

64785, 37901, 168349, 200900, 357469,

400928, 786594, 836493, 207046, 549038,

600400, 859427, 920001, 900046.

NOTATION AND NUMERATION OF LARGE NUMBERS

To make it easy to read large numbers, the digits in which they are written are separated into groups with three digits in each, beginning at the *right*. Each group is called a **period** and is usually separated from the other periods by the use of the comma. Each period has a name. From the right the first period is **units,** the second **thousands,** the third **millions.** In *each period* the first digit from the right is *units,* the second *tens,* and the third *hundreds*. Each digit has, therefore, two values—its own, or its *intrinsic* value, and its *local* value, which is given to it by the *place* it occupies. For example, in 964 the intrinsic value of 9 is nine, but, as it occupies the third place from the right, its *local* value is 9 *hundred*.

Study the following table:

Periods	billions	millions	thousands	hundreds
Places	hundred billions ten billions billions	hundred millions ten millions millions	hundred thousands ten thousands thousands	hundreds tens units

6 8 2, 9 7 5, 3 2 9, 6 4 7.

The number is 682 billion 975 million 329 thousand 647.

What is the *local* value of 975? of 64? of 8? of 753? of 7532? of 8297?

EXERCISE 3

1. Read the following numbers:

(a) 865, 407, 829, 630, 4652, 4092, 7001, 9830, 9400.

(b) 10000, 12004, 15603, 54987, 68436, 20093, 99827.

(c) 192628, 600503, 417110, 982157, 468392, 948069.

(d) 1000000, 1230000, 2345400, 3684750, 9847326.

2. What is the general name for the three digits in the first period? in the second period? in the third period?

3. Divide the number 543212789 into periods and then read it. Read the second period from the right, giving it its name. Read the first. Read the third.

4. Express the following numbers in figures: nine hundred and four thousand six hundred and eighty; eighty million fifty thousand seven hundred and nine; forty thousand and ten; three hundred and three thousand five hundred and twenty-eight; two hundred and three million seven hundred thousand and eighty-seven.

5. Read the following:

(a) The production of Canadian coal in 1919 was as follows: Nova Scotia, 5,702,316 tons; New Brunswick, 177,976 tons; Saskatchewan, 381,967 tons; Alberta, 4,983,730 tons; British Columbia, 2,429,211 tons; Yukon, 1,100 tons. Total, 13,676,300 tons.

(b) In 1916 the Canadian Northern Railway carried 7,574,500 sacks of flour, 131,978,809 bushels of grain, 488,809 head of live stock, 1,741,031 tons of coal, and 286,745 cords of firewood.

(c) The mineral production in Canada for the year 1916 was as follows:

Nova Scotia $19,963,985; New Brunswick $878,446; Quebec $14,397,909; Ontario $80,379,352; Manitoba $1,819,921; Saskatchewan $583,708; Alberta $13,336,-702; British Columbia $40,191,744; and the Yukon $5,805,687. (Note: $ stands for dollars.)

(d) In 1916 Canada produced $352,543,470 of farm products, $51,271,400 of forest products, $22,377,977 of fish, and $242,034,998 of manufactured products.

ROMAN NUMERALS

The symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, are called **Arabic Numerals,** but in writing numbers other symbols called **Roman Numerals** are sometimes used. These are the capital letters I, V, X, L, C, D, and M. These numerals are used in putting dates on buildings, in numbering the chapters of a book, in numbering the hours on the dials of clocks, and for other special purposes.

The value of I is 1; of V, 5; of X, 10; of L, 50; of C, 100; of D, 500; and of M, 1000.

All numbers are expressed in these numerals as follows:

(*a*) The letters I, X, C, and M, may each be repeated as often as three times, and when the letter is repeated its value is repeated. Thus XX represents 2 tens, that is 20; MMM represents 3 thousand, that is 3000.

(*b*) A letter of less value placed *after* a letter of greater value *adds* its value to the greater. Thus XVII is 10 and 5 and 1 and 1, or 17.

(*c*) I placed *before* V or X takes away its value from that of V or X. Thus IV is 4 and IX is 9. X placed before L or C takes away its value from that of L or C. Thus XL is 40 and XC is 90. So, too, C placed before D or M takes away its value from that of D or M. Thus CD is 400 and CM is 900.

(*d*) Placing a bar over a letter changes its value into thousands. Thus $\overline{D}$ is 500 thousand, that is, 500,000.

EXERCISE 4

(Oral and Written)

1. Read the following and then write each in Arabic numerals:

III, XXX, MM, CC, VII, XXVI, LXXVIII, CLV,
CCCXXIII, DCLXI, MMDCXXVII, IV, IX, XL, XC,
CD, CM, XIV, XXIX, XLIV, XCIX, CDXLVI,
CMLXXIV, MMCDLXXXIV.

2. Express in Roman numerals:

24, 43, 79, 81, 63, 68, 272, 504, 813, 319, 682, 879, 1746,
3947.

3. Read the following and write in Arabic numerals:

XII, XVIII, XXIV, XXVI, XXXIV, XXXIX, LIX,
LVI, LXIV, XCVIII, CIX, CXXVII, CXLVI, DC,
MCM, MCMX, MMCDLIX, MMMCMXLIV.

4. Express in Roman numerals:

18, 36, 48, 49, 54, 58, 66, 69, 74, 86, 89, 94, 96, 99, 119,
133, 134, 139, 187, 236, 475, 523, 684, 739, 815, 936,
998, 1005, 1487, 1896, 2054, 3865.

5. Read and write in Arabic numerals:

XCI, MCDXLIV, LXIX, CCXIX, XXXVIII, MMMI,
XVI, DCCCXCIX, CDIV, MMCMXCIX, XXIV,
CXVI, DCXLI, CMX.

6. Write in Roman numerals the number of:

The year in which you were born; the year in which
your school-house was built; the year in which our first
Dominion Day was celebrated; the year in which Columbus
discovered America; the year in which the present King
began his reign.

ADDITION

The sign + is called **plus** and is used to indicate that
the numbers between which it is placed are to be *added*.

The sign = is used for the word "equal" or "equals."
Thus 27 + 5 = 32 is read 27 plus 5 equals 32.

Addition is the method or process by which *two or more* numbers are united or combined to form *one* number.

The numbers to be united are called **addends,** and the one number obtained by uniting them is called their **sum.**

What is the sum of 3 dollars and 4 dollars? of 3 pounds and 4 pounds? of 3 trees and 4 trees? of 3 trees and 4 pounds? of 3 pounds and 4 dollars? of 3 pens and 4 rings?

Like numbers are those which have the same name or **denomination,** that is, they are numbers which are obtained by using the same *unit*.

It is evident that only *like* numbers can be added. What is the denomination of the sum? In written exercises in addition, *units* are usually placed under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on. Why?

ORAL EXERCISE

1. The following table contains 45 different combinations of numbers from 1 to 10. These are known as the 45 "primary facts" of addition. Practise on the table until you can name any of the sums the moment your eye rests upon the numbers to be added:

1	2	2	3	3	4	3	4	5	4	5	6	4	5	6
1	1	2	1	2	1	3	2	1	3	2	1	4	3	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

7	5	6	7	8	5	6	7	8	9	6	7	8	9	6
1	4	3	2	1	5	4	3	2	1	5	4	3	2	6
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

7	8	9	7	8	9	7	8	9	8	9	8	9	9	9
5	4	3	6	5	4	7	6	5	7	6	8	7	8	9
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

2. Practise adding the following until you can add the groups in any order at the rate of 60 groups a minute:

2	9	8	5	9	9	5	3	2	7	2	8	7	8	7
5	1	8	6	1	1	1	2	7	2	4	7	5	7	4
6	9	8	4	4	6	5	6	2	5	5	7	9	9	7

8	7	8	9	6	5	4	7	5	9	4	8	7	9	7
9	3	9	2	7	3	7	2	4	9	1	1	6	4	3
8	8	9	6	6	5	8	7	5	6	6	6	9	9	4

4	8	5	4	6	9	1	2	2	9	8	3	7	8	7
4	1	6	3	2	1	6	3	6	1	4	7	6	4	6
4	7	5	9	6	1	7	4	2	8	8	1	7	9	3

6	9	4	3	2	3	9	7	3	6	9	8	9	8	7
6	4	2	6	4	5	2	3	2	5	2	7	7	3	7
6	5	9	8	7	3	9	6	8	8	8	8	7	9	5

4	2	8	8	9	6	4	8	4	6	2	5	5	7	6
4	8	7	3	9	9	1	5	4	6	9	3	5	6	6
9	8	6	5	9	8	4	5	2	6	7	7	5	2	6

3. On the table below practise as follows:

(a) In each column begin at the top and add the first and second numbers; then the second and third; then the third and fourth; and so on.

(b) Add the first, second, and third numbers; then the second, third, and fourth; then the third, fourth, and fifth; and so on.

(c) Continue as above, by adding four numbers, then five, and so on until the whole column is added.

(d) Repeat the practice, beginning at the bottom instead of the top.

1	2	1	2	1	2	1	2	3	3
2	1	1	1	2	2	3	3	1	2
1	2	3	2	1	3	1	1	2	2
2	3	1	2	3	2	1	3	2	3
2	4	3	2	4	4	4	1	2	4
3	1	3	4	2	1	4	3	4	1
4	5	2	2	5	5	2	4	1	5
5	3	6	4	5	2	4	3	6	3
5	6	3	6	1	7	5	6	6	5
7	4	6	5	7	7	3	7	4	6
6	3	8	2	5	1	8	4	8	2
9	7	8	9	3	8	8	9	9	8
8	9	5	7	9	6	5	9	4	7
6	5	3	6	8	8	9	7	9	3

4. Beginning first at 0, then at 1, then at 2, and so on up to 10, add by twos, by threes, by fours, by fives, by sixes, by sevens, by eights, by nines, and by tens, until the sum in each case is greater than 100.

EXERCISE 5

1. Add:

(a) 3754
    2862
    1457
    ———

(b) 20234
    683
    4965
    68
    ———

(c) 275 acres
   146 "
    27 "
   ———

2. In Example 1 (b) above, what figures represent thousands? units? ten thousands?

3. Add 5732, 6721, 3466, 4269, 6535.

4. Add 2768, 5329, 4605, 475, 16.

5. Add 375, 506, 258, 327, 580, 647, 846.

6. Add 625, 494, 742, 673, 574, 654, 638.

7. Add without arranging in columns:
    (a) 564, 683, 684, 502, 376, 726, 877.
    (b) 436, 47, 449, 498, 736, 274, 888.
    (c) 4671, 272, 45, 7, 625.

8. From the following time sheet giving the number of hours of work for six carpenters in one week, find the total number of hours each man worked, the number of hours all worked each day, and the number of hours all worked in the week:

Name	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Total
Jno. Jackson	8	9	7	9	8	6	
Geo. Laing	5	9	9	7	7	4	
Wm. Graham	9	8	8	6	9	5	
Jas. Storey	7	9	9	8	7	6	
Robt. Watson	3	8	9	7	8	3	
Thos. Grover	8	7	8	5	9	4	
Total							

9. Without arranging in columns, find the sum of:

(a) 53 + 74 + 42

(b) 29 + 63 + 49 + 36

(c) 26 + 32 + 4 + 17 + 9

(d) 327 + 295 + 74 + 482 + 6

(e) 6537 + 491 + 836 + 2964 + 579

CANADIAN MONEY

The sign $ stands for the word **dollars**, and the letter c. stands for **cents.** Thus 17c. is read 17 cents, and $15 is read 15 dollars.

When dollars and cents are written together, the cents are separated from the dollars by a point, thus (·). Thus $42 and 58c. is written $42·58.

When the number of cents is less than 10, a cipher must occupy the first place at the right of the point. Thus $8 and 4c. is written $8·04. Why? In arranging numbers expressing dollars and cents for adding, the dollars must be placed under the dollars and the cents under the cents. Why? The points will then stand under one another.

Thus:

(a) $376·84	(b) $4397·03	(c) $ 53·70
43·09	69·50	786·10
706·40	240·84	9·08

EXERCISE 6

1. Read the following:

$2·25, $3·50, $5·04, $6·00, $15·13, $20·04, $0·65.

2. Write the following in figures:

One dollar and ninety-three cents, two dollars, two dollars and five cents, two dollars and fifteen cents, three dollars and twenty-five cents, three dollars and sixty-seven cents, three dollars and ninety-five cents, four dollars, four dollars and thirty-seven cents, five dollars.

3. Find the totals of the following sums of money:

(a) $3·50	(b) $4·75	(c) $3·87	(d) $2·91
·37	2·67	4·25	5·42
1·25	3·85	·89	6·62
2·65	4·00	·46	7·38
		5·38	2·25
			9·09

2

(e) $4·25	(f) $8·00	(g) $5·13	(h) $9·08
3·06	2·05	6·85	·75
7·00	6·10	1·08	·92
8·15	·05	·63	·68
·38	·09	·75	1·05
·57	3·46	4·56	3·15
9·02	2·12	·10	8·95

4. Find the totals of the following sums of money:

   (a) $6·05, $3·08, 57c., 5c.

   (b) $2·00, $6·81, 65c., 4c., 17c.

   (c) $4·02, 75c., 89c., 3c., $4·59, 28c.

—(d) $6·15, $3·82, $2·95, 15c., 38c., 5c., $1·08.

◁(e) $7·28, $9·03, $6·57, 40c., 9c., $1·28.

5. Find the totals of the following:

   (a) Three dollars and fifteen cents, sixty-seven cents, nine cents, forty-five cents.

   (b) Four dollars and five cents, two dollars and twenty-five cents, twenty-four cents, eight cents, one dollar and six cents.

   (c) Seven dollars and three cents, five dollars and forty cents, two dollars and twenty cents, sixty-five cents, seven cents.

6. Read, arrange, and add the numbers in each of the following sets:

   (a) $4·75, $3083·09, $72·50, $9·32, $384.

   (b) $93·48, $406·30, $8·07, $5709·80.

   (c) $500, $93·05, $364·05, $47·09.

Accuracy is of the greatest importance. All answers should be tested or checked. This is done by going over the work a *second* time, but, if possible, in a way different from that taken the *first* time. To check addition, first add the columns up, and then add down. Inaccuracy is often caused by badly formed figures, lack of neatness, and improper arrangement.

EXERCISE 7
(Rapid Drill and Accuracy)

Add rapidly up and then down:

(1)	3678	(2)	8765	(3)	4578	(4)	7841	(5)	92438
	9765		4321		3287		8265		56789
	3146		3456		9976		9874		29475
	8973		7897		4521		3762		86794
	7695		6543		7894		9876		98764
	1878		7896		3786		8765		57634
	6543		8342		7312		4267		78929
	2109		8976		9543		2894		———

(6)	38567	(7)	87653	(8)	98947	(9)	2789	(10)	8276
	23456		67497		36584		5647		4857
	87564		82189		28796		4786		2964
	34567		47635		47863		5938		3845
	89756		54986		28947		2749		8794
	93564		38275		59638		4638		4683
	46289		49586		28647		2947		2957
	———		———		———		4685		4683
							9376		2947

(11)	$426·89	(12)	$28913·46	(13)	$ 429·05
	107·64		5264·73		376·79
	963·66		84927·82		518·43
	478·23		368·46		792·58
	862·78		93758·49		4329·04
	478·42		8473·26		6521·37
	338·25		82594·38		9627·58
	493·86		68·49		48·36
	———		5·63		25·13

ORAL EXERCISE

1. What is meant by "cash"? "change"? "cashier"? "customer"?

2. Explain how a cashier, salesman, or saleslady makes up the right amount of change to be given to a customer.

3. Using the coins and bank bills of the greatest value possible in each case, "make change," orally, for customers purchasing and paying as follows:

	Purchase	Payment		Purchase	Payment
(a)	26c.	50c.	(n)	$1·67	$ 1·75
(b)	$1·15	$2·00	(o)	3·62	4·00
(c)	2·11	3·00	(p)	2·98	5·00
(d)	4·08	4·50	(q)	4·06	5·00
(e)	89c.	2·00	(r)	4·09	6·00
(f)	1·38	2·00	(s)	5·28	5·50
(g)	1·64	2·00	(t)	5·75	7·00
(h)	2·15	3·00	(u)	7·39	10·00
(i)	3·49	5·00	(v)	5·63	10·00
(j)	4·18	5·00	(w)	7·62	10·00
(k)	1·72	5·00	(x)	4·44	10·00
(l)	58c.	2·00	(y)	6·69	10·00
(m)	1·31	5·00	(z)	8·21	10·00

SUBTRACTION

ORAL EXERCISE

In the following examples state quickly what number, in each case, must be *added* to the *lower* number in order that the *upper* number may be the sum:

1.	9	8	7	10	13	12	15	11	16	10
	3	5	2	7	4	6	8	6	9	4

2.	14	10	12	18	11	10	16	13	11	15
	7	5	3	9	7	1	8	4	2	6

3.	12	17	13	14	11	16	13	14	10	12
	5	9	5	6	3	7	7	9	2	4

4.	15	25	65	85	25	45	55	45	75
	9	9	9	9	19	19	19	29	39

5.	14	34	34	64	94	42	62	82
	8	8	18	28	38	5	35	25

6.	36	42	81	54	65	71	94	40
	17	14	32	37	26	34	29	24

7. If, in each example above, the lower number were taken out of, or away from, the upper number, *what number* would be left?

8. What number, added to 2, will give 100? 98? 96? 94? and so on. Begin at 100 and count backward by *twos*.

9. Begin at 100 and count backward by threes, by fours, and so on up to nines.

10. Begin at 97 and count backward by twos, by threes, by fours, and so on.

11. Begin at 98 and count backward by threes, by fours, and so on.

The number which, added to one number, makes another, is called the **difference** between the two numbers. Thus, 8 is the difference between 7 and 15 because 8 added to 7 makes 15.

The number which is left when one number is taken out of, or away from, another number, is called the **remainder**. Thus, when 7 is taken away from, or, as it is usual to say, **subtracted** from 15, the remainder is 8.

It is evident that difference and remainder are really two names for the same thing.

The process, or method, of finding the difference between two numbers, or of finding the remainder when one number is *subtracted* from another, is called **subtraction.**

The larger of the two numbers in subtraction is called the **minuend,** the smaller the **subtrahend.**

The minuend is the sum of the remainder and the subtrahend.

The sign for subtraction is − and is called the **minus** sign. It indicates that the number after it is to be subtracted from the number before it. Thus, 62 − 25 is read 62 *minus* 25 and means that 25 is to be subtracted or taken away from 62.

The numbers are arranged as for addition, the smaller being placed under the larger.

Subtraction is only another form of Addition, for in order to find the difference between two numbers it is only necessary to find the number which must be *added* to the smaller to give the larger.

EXERCISE 8

Subtract and check:

1. 89   786   674   63   84   70
    53   45   52   27   39   24
    ──   ───   ───   ──   ──   ──

2. 456   975   578   826   500   807   353   764
    128   246   183   458   167   248   168   532
    ───   ───   ───   ───   ───   ───   ───   ───

3. 6982   7026   6304   5500   8306   6304   9000
    5431   4007   2012   2870   7029   2012   8023
    ────   ────   ────   ────   ────   ────   ────

4. 3245   8164   2001   7000   6111
   1679   3275   1009   6006   4789
   ‾‾‾‾   ‾‾‾‾   ‾‾‾‾   ‾‾‾‾   ‾‾‾‾

5. $80·00 $90·00 $50·00 $40·00 $75·00 $60·00
    47·16   37·28   16·59   26·54   39·52   47·64
   ‾‾‾‾‾‾   ‾‾‾‾‾‾   ‾‾‾‾‾‾   ‾‾‾‾‾‾   ‾‾‾‾‾‾   ‾‾‾‾‾‾

6. $100·00 $200·00 $500·00 $600·00 $900·00
    65·37   134·68   256·58   381·57   489·31
   ‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾

7. $1000·00 $427·39 $891·69 $748·39 $823·15
     652·68   168·57   295·71   299·42   284·68
   ‾‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾   ‾‾‾‾‾‾‾

8. Find the difference between 64805 and 37286; 59420 and 76905; 86056 and 272421.

9. Find the difference between 6037 − 586 and 4910 − 238.

10. From 5637 − 496 take 4123 − 1235.

MULTIPLICATION

How many apples are 4 apples + 4 apples + 4 apples?

🍎🍎🍎🍎   🍎🍎🍎🍎   🍎🍎🍎🍎
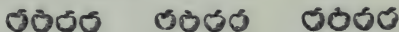
Here 4 apples are *repeated* 3 times, and the result is 12 apples, which is expressed thus: 3 *times* 4 apples are 12 apples.

Make pictures showing: 2 *times*, or twice, 6 leaves; 6 *times* 3 eggs; 5 *times* 2 trees; 4 *times* 5 tops.

How many are two times 6? 6 times 3? 5 times 2? 4 times 5? How is the result in each case obtained? In the same way find 4 times 687, thus:

$$
\begin{array}{r}
687 \\
687 \\
687 \\
687 \\
\hline
2748
\end{array}
$$

Instead of adding here, the result could be obtained more quickly by using a *4-times* **table,** that is, by finding and then *memorizing* the results of 4 times 1, 4 times 2, 4 times 3, and so on up to 4 times 9. For in the first column of the addition problem there are 4 times 7; in the second column 4 times 8 tens; and in the third column 4 times 6 hundred.

This second process is called **multiplication.** What two methods can be employed to find 6 times 529?

Multiplication is the method, or process, in which certain memorized addition results are used to find the sum obtained by repeating one number, as an addend, as often as there are units in another number.

The repeated number is called the **multiplicand.** The number indicating the *number of times* the multiplicand is repeated is called the **multiplier.** The result obtained in multiplication is called the **product.**

The sign of multiplication is ×. Thus, 14 × 4 is read 14 *multiplied* by 4, or 4 *times* 14, or 14 *times* 4.

Read in three ways: 13 × 6, 18 × 9, and 16 × 7.

Find by addition: 2 times 1, 2 times 2, and so on up to 2 times 12.

This forms the **multiplication table** of 2 *times.*

In the same way, make tables for 3 times, 4 times, and so on up to 12 times.

All the tables will be found in the following:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

These tables should be so thoroughly committed to memory that answers will be given *without hesitation* to any of these four types of exercises:

$$(a)\ 5 \times 9 = ?$$
$$(b)\ ? \times 9 = 45$$
$$(c)\ 5 \times ? = 45$$
$$(d)\ ? \times ? = 45$$

1. Answer quickly:

3×2	4×7	6×5	4×9	7×3
9×8	6×4	2×8	12×3	9×7
6×6	7×6	12×9	9×3	7×8

2. Answer quickly:

$? \times 4 = 36$	$? \times 7 = 49$	$? \times 3 = 12$
$? \times 5 = 35$	$? \times 12 = 96$	$? \times 8 = 48$
$? \times 9 = 54$	$? \times 10 = 30$	$? \times 6 = 24$

3. Answer quickly:

$7 \times ? = 28$	$12 \times ? = 84$	$5 \times ? = 40$
$6 \times ? = 36$	$9 \times ? = 72$	$4 \times ? = 24$
$8 \times ? = 56$	$7 \times ? = 35$	$3 \times ? = 18$

4. Answer quickly:

$? \times ? = 42$	$? \times ? = 30$	$? \times ? = 27$
$? \times ? = 108$	$? \times ? = 55$	$? \times ? = 63$
$? \times ? = 49$	$? \times ? = 72$	$? \times ? = 36$

5. Multiply quickly each digit in 93796785 by 2, 4, 6, 7, 8, and 9.

6. What are 8 pencils worth at 5 cents each?

7. A Public School Arithmetic costs 10 cents. What will 6 of them cost?

8. What are 12 chairs worth at $7 each?

9. If sugar is worth 8c. a pound, find the value of nine pounds.

10. How far can a boy ride on his bicycle in 4 hours if he rides at the rate of 7 miles an hour?

11. There are 8 pints in one gallon. How many pints are there in 5 gallons?

12. What would 6 pairs of boots cost at $8 a pair?

13. A boy planted 7 rows of potatoes, each 9 rods long. What was the length of all the rows?

14. What is the price of 3 pairs of hockey boots at $6 a pair?

15. What is the value of 8 pairs of skates at $6 a pair?

The work in multiplication is usually set down thus:

$$\begin{array}{r} 483 \\ 6 \\ \hline 2898 \end{array}$$

Here 483 is the multiplicand, 6 the multiplier, and 2898 the product.

EXERCISE 9

1. Multiply 6893 by 2, by 4, by 6, and by 8.

2. Multiply 4715 by 3, by 5, by 7, and by 9.

3. Multiply 3097 by 3, by 4, by 5, by 7, and by 9.

4. Find the product of eighty-seven thousand four hundred and six by 4, by 8, and by 9.

5. Multiply eighty-nine thousand eight hundred and fifty-seven by 3, by 4, by 5, by 6, by 7, and by 9.

6. Multiply 75087 by 8, by 7, and by 12.

7. Multiply 83009 by 9, by 7, and by 6.

8.

$ 3·10	$ 4·67	$ 5·32	$ 6·53	$ 7·63	$ 8·72
8	3	6	7	5	9

$ 9·48	$12·57	$13·65	$14·63	$15·78	$20·36
8	5	7	9	4	3

$23·69	$34·65	$49·58	$69·53	$62·75	$73·59
8	9	6	11	12	10

34123	42561	48763	54309	67482	52637	40519
8	5	11	9	7	4	12

30026	67849	59768
10	6	9

9. Multiply each of the following numbers by 10: 93789, 408537, 39207.

10. Without multiplying, write at once the answers to the following:

527×10, 38649×10, 4005×10, 8030×10.

ORAL EXERCISE

Give quickly the products of:

30×8	50×7	60×9	80×6	70×4
40×8	20×7	10×9	100×6	31×4
52×8	51×7	53×9	43×6	42×4
44×8	64×7	62×9	61×6	75×4
73×8	71×7	74×9	85×6	83×4
82×8	84×7	81×9	96×6	95×4
93×8	92×7	91×9	76×6	86×4
36×8	46×7	26×9	103×6	105×4
27×8	56×7	47×9	68×6	77×4
91×4	72×6	76×7	84×11	19×8
57×3	48×7	57×8	39×6	17×9
64×9	36×10	96×2	40×5	16×6
26×12	18×8	13×11	27×4	14×7

EXERCISE 10

(Oral and Written)

1. How many times 2 must be added to 3 times 2 to make 5 times 2?

$$
\begin{array}{cccc}
\bullet & \bullet & \bullet & \\
& & & = 3 \text{ times } 2;
\end{array}
\quad
\begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet
\end{array}
= 5 \text{ times } 2.
$$

2. How many times 3 must be added to 2 times 3 to make 6 times 3? Use dots to find this out.

3. How many times 4 must be added to 3 times 4 to make 7 times 4?

4. How many times 9 must be added to 3 times 9 to make 8 times 9?

5. How many times 15 must be added to 3 times 15 to make 10 times 15?

6. How many times 25 must be added to 10 times 25 to make 20 times 25? Find 10 times 25. Then find 20 times 25.

7. How many times 33 must be added to 10 times 33 to make 20 times 33? Find 10 times 33. Then find 20 times 33.

8. How many times 48 must be added to 10 times 48 to make 20 times 48? Find 10 times 48. Then find 20 times 48.

9. In what other way besides adding them can the sum of these numbers be found:

| 150 | 250 | 330 | 480 |
150	250	330	480

10. Find 20 times 15, 20 times 25, 20 times 33, and 20 times 48 in this new way.

11. Find 20 times 38, 20 times 46, 20 times 67, 20 times 78, and 20 times 89.

12. Multiply 463, 827, 958, 3456, 4265, by 20.

The work is set down thus:
```
   463
    20
  ____
  9260
```

13. How often must 10 times 53 be taken to make 30 times 53? Find 10 times 53, then find 30 times 53.

14. Multiply 68, 97, 492, 634, 768, 347, 4568, 5876, by 30. Set down work as above.

15. How often must 10 times 64 be taken to make 40 times 64?

16. Multiply 376, 429, 685, 5873, 4967, by 40. Set down work as above.

17. How often must 10 times 76 be taken to make 50 times 76? to make 60 times 76? to make 70 times 76? to make 80 times 76? to make 90 times 76?

18. Multiply 76 by 50, by 60, by 70, by 80, and by 90.

19. Multiply 927 by 50, by 60, by 70, by 80, and by 90.

20. Multiply 4853 by 50, by 60, by 70, by 80, and by 90.

ORAL EXERCISE

Find the product quickly of:

1. 6×40; 7×50; 9×80; 12×60; 11×90; 8×70.

2. 30×10; 40×10; 50×20; 60×30; 40×40; 50×50.

3. 48×30; 65×20; 75×40; 82×60; 70×50; 74×20.

4. 33×40; 64×30; 82×60; 95×20; 45×30; 85×40.

EXERCISE 11

(Oral and Written).

1. How many times 35 must be added to 10 times 35 to make 13 times 35?

Find 10 times 35 and 3 times 35, then 13 times 35.

2. How many times 35 must be added to 10 times 35 to make 14 times 35?

Find 10 times 35 and 4 times 35, then 14 times 35.

3. Find 15 times 35, 16 times 35, 17 times 35, 18 times 35, and 19 times 35.

4. How many times 35 must be added to 20 times 35 to make 24 times 35? Find 20 times 35 and 4 times 35, and then 24 times 35.

The work is set down thus:

$$
\begin{array}{r}
35 \\
24 \\
\hline
140 \\
700 \\
\hline
840
\end{array}
$$

The 0 in the second product is usually omitted.   Why?

5. Find 23 times 35, 27 times 35, 28 times 35.

6. Multiply 489 by 21, 25, and 29.   The work is set down thus:

$$
\begin{array}{r}
489 \\
21 \\
\hline
489 \\
9780 \\
\hline
10269
\end{array}
$$

7. Multiply 6834 by 31, by 34, by 36, by 39, by 42, by 45, by 47, and by 48.   Set down work as above, and omit the unnecessary 0's.

8. Multiply 4279 by 53, by 55, by 57, by 62, by 68, by 73, and by 75.

9. Multiply 8596 by 81, by 89, by 76, by 94, and by 95.

EXERCISE 12

1. How many times must we take 10 times 43 to have 100 times 43?   Find 100 times 43.

2. Find 100 times 68, 100 times 79, 100 times 86.

3. How many times must we take 100 times 57 to have 200 times 57?   Find 200 times 57.

4. Find 300 times 57, 400 times 57, 500 times 57.

5. Multiply 863 by 100, by 200, by 300, by 400, by 500, by 700, and by 900.

6. Multiply 7491 by 200, by 400, by 500, by 600, by 800, and by 900.

The work is set down thus:

$$
\begin{array}{r}
7491 \\
200 \\
\hline
1498200
\end{array}
$$

7. Find the value of 600 lb. of beef at 32c. a lb.

8. Find the value of 800 lb. of pork at 29c. a lb.

9. Find the value of 700 lb. of lamb at 37c. a lb.

10. Find the value of 900 lb. of fish at 18c. a lb.

11. Find the products quickly of:

9×300	12×600	11×800	8×500	9×700
6×400	15×200	20×300	30×600	40×900
22×500	33×600	41×700	60×900	72×400

EXERCISE 13

1. Show that 126 times 75 = 100 times 75 + 20 times 75 + 6 times 75.

Find 126 times 75.

The work is set down thus:

$$
\begin{array}{r}
75 \\
126 \\
\hline
450 \\
1500 \\
7500 \\
\hline
9450
\end{array}
$$

The numbers 450, 1500, 7500, which, added together, give the product, are called **partial products.** Observe that in the second partial product, the last 0, and in the third partial product, the last two 0's, may be omitted. Why?

2. Find 138 times 64, 147 times 82, 159 times 93. Set down work as above and omit unnecessary 0's.

3. Multiply 6546 by 126, by 234, by 405, and by 563.

4. Multiply 4987 by 653, by 748, by 921, by 465, and by 792.

5. Multiply 5369 by 498, by 651, by 796, by 583, and by 471.

6. Multiply 9832 by 532, by 674, by 891, by 758, and by 496.

7. Multiply 8173 by 689, by 546, by 709, by 803, and by 604.

8. Multiply 9756 by 427, by 865, by 407, by 389, and by 706.

Numbers which when multiplied together make another number are said to be the **factors** of that number. Thus 6 and 4 are factors of 24; 2 and 15 are factors of 30; 3, 2, and 7 are factors of 42.

ORAL EXERCISE

1. Supply the missing factors:

$4 \times ? = 16$	$? \times 8 = 56$	$? \times ? = 18$	$? \times ? \times ? = 48$
$5 \times ? = 35$	$? \times 7 = 28$	$? \times ? = 96$	$? \times ? \times ? = 36$
$9 \times ? = 72$	$? \times 2 = 18$	$? \times ? = 36$	$? \times ? \times ? = 108$
$7 \times ? = 42$	$? \times 11 = 66$	$? \times ? = 54$	$? \times ? \times ? = 28$
$8 \times ? = 40$	$? \times 5 = 30$	$? \times ? = 20$	$? \times ? \times ? = 100$
$6 \times ? = 48$	$? \times 3 = 21$	$? \times ? = 63$	$? \times ? \times ? = 210$

DIVISION

ORAL EXERCISE

1. How much is 3×5? What number has to be repeated 5 times to make 15?

2. How much is 8×7? What number has to be repeated 7 times to make 56? What number should be put in this blank: $\ldots \times 7 = 56$?

3. Name the number which should be written in each of these blanks:

$\ldots \times 4 = 20;$ $\ldots \times 9 = 63;$ $\ldots \times 10 = 60;$
$\ldots \times 7 = 35;$ $\ldots \times 6 = 42;$ $\ldots \times 3 = 27;$
$\ldots \times 5 = 40;$ $\ldots \times 2 = 14;$ $\ldots \times 9 = 54;$
$\$\ldots \times 6 = \$24;$ $\$\ldots \times 8 = \$72;$ $\$\ldots \times 7 = \$49;$
$\ldots$ feet $\times 9 = 45$ feet; $\ldots$ pints $\times 2 = 8$ pints;
$\ldots$ hats $\times 6 = 60$ hats; $\ldots$ hundred $\times 5 = 45$ hundred;
$\qquad \ldots$ thousand $\times 4 = 28$ thousand.

4. What sum must be repeated 6 times to give $24? If 24 dollars are divided equally among 6 boys, how many dollars will each boy get? What is the *sixth* part of 24 dollars?

5. How many oranges must be taken 5 times to make 30 oranges? If 30 oranges are divided equally among 5 boys, how many oranges will each boy get? What is the *fifth* part of 30 oranges?

6. If 54 cents are divided equally among 9 girls, how many cents will each girl get? What is the *ninth* part of 54 cents?

7. If a boy walks 12 miles in 3 hours, how far will he walk in 1 hour? What is the *third* part of 12 miles?

8. What is the half of 8? The third part of 15? The seventh part of 56? The eighth part of 48? The tenth part of 30? The ninth part of 63?

9. Name the numbers which should be written in these blanks:

$$5 \times 6 = 6 \times \ldots; \qquad 9 \times 8 = 8 \times \ldots;$$
$$7 \times 4 = 4 \times \ldots; \qquad 3 \times 5 = 5 \times \ldots;$$
$$2 \times 9 = 9 \times \ldots.$$

10. 4 times $6 is how many times $4? 5 times 7 men is how many times 5 men? 8 times 7 feet is how many times 8 feet?

11. What number must be taken 3 times to make 24? How many 3's are there in 24? Why?

12. What sum taken 9 times makes $54? Then $54 is how many times $9? Why?

13. What quantity must be taken 6 times to give 30 quarts? How many times does 30 quarts contain 6 quarts? Why?

14. 56 feet is equal to 8 times what length? How often does 56 feet contain 8 feet? Why?

15. How often does 18 contain 3? 21 contain 7? 72 contain 9? 36 contain 4? 25 contain 5? 42 contain 6?

16. How many 5's are there in 20? 7's in 28? 9's in 27? 6's in 48? 8's in 64? 3's in 18? 2's in 14? 10's in 60? 4's in 32?

17. What distance must be taken 3 times to give 12 miles? 4 miles $\times$ 3 = 3 miles $\times$ ....? Then 12 miles is equal to how many times 3 miles? If a boy walks 3 miles an hour, how long will it take him to walk 12 miles?

18. $56 is 7 times what sum? $8 $\times$ 7 = $7 $\times$ ....? Then how often does $56 contain $7? If $56 is given to a number of boys so that each boy gets $7, how many boys are there?

19. If a pound of sugar can be bought for 8 cents, how many pounds can be bought for 40 cents?

20. How many times is $3 contained in $12? in $15? in $13? in $21? in $18? in $16?

21. How many times is 9 contained in 36? in 45? in 43? in 38? in 40?

22. How often is 8 contained in 35? Then $35 = \ldots \times 8 + \ldots$, also $35 = 8 \times \ldots + \ldots$

23. Name the numbers which should be written in these blanks:

$$54 = \ldots \times 7 + \ldots$$
$$62 = \ldots \times 9 + \ldots$$
$$23 = \ldots \times 3 + \ldots$$
$$47 = \ldots \times 6 + \ldots$$
$$18 = 8 \times \ldots + \ldots$$
$$38 = 12 \times \ldots + \ldots$$
$$7 = 2 \times \ldots + \ldots$$
$$75 = 9 \times \ldots + \ldots$$

24. The product of two numbers is 35. One of the numbers is 7. What is the other?

25. The product of two numbers is 12. One of the numbers is 6. What is the other?

26. The product is 28. One of the numbers is 4. What is the other?

27. The product is 48. One of the numbers is 8. What is the other?

It is clear that the answers for the problems in the oral exercise above are found by making use of the multiplication tables. The examples, however, are not exactly like those given in multiplication. In multiplication *two factors* are given, and their *product* has to be found. In this oral exercise it will be seen that the *product* and *one factor* are given, and the *other factor* has to be found. The process by which this factor is found is called **division.**

Division is the process of finding *what number* is contained a given number of times in another, and (since *4 times* 3 is equal to *3 times* 4) it is also the process of

finding *how many times* one number is contained in another.

Division is indicated by writing the sign ÷ between the numbers, thus:

8 ÷ 2 means "8 *divided* by 2," the problem being to find the *number* which taken *2 times* makes 8, or to find the *number of 2's* in 8.

In the Example, 28 ÷ 7, the answer is 4. In division the number to be divided, 28, is called the **dividend,** the given factor, 7, is called the **divisor,** and the answer, 4, is called the **quotient.**

If the dividend is not exactly equal to the product of the quotient and divisor, there will be a **remainder.** Thus, in 23 ÷ 4, it will be seen that $23 = 5 \times 4 + 3$. Here 3 is the remainder.

Division is also indicated by writing the dividend above and the divisor below a horizontal line. Thus:

$$\frac{8}{2} \text{ means } 8 \div 2.$$

What is the corresponding term in multiplication for the dividend? the quotient? the divisor?

By what other process could the *number of 8's* in 37 be found? To what terms in this process does the dividend correspond? the divisor? the remainder?

Practise these exercises in two ways:

(*a*) State *quotients only*, thus, for (1) say 12, 4, 8, etc.

(*b*) Give answers, thus, 144 = 12 times 12, or 12 twelves; 48 = 12 times 4, or 4 twelves; 96 = 12 times 8, or 8 twelves, etc.

A

State the quotients rapidly:

1. 144, 48, 96, 36, 120, 72, 24, 132, 84, 108, 60 ÷ 12
2. 99, 27, 54, 90, 108, 18, 63, 81, 36, 72, 45 ÷ 9
3. 72, 48, 24, 96, 16, 80, 40, 32, 88, 56, 64 ÷ 8
4. 70, 14, 49, 63, 77, 56, 28, 84, 21, 42, 35 ÷ 7
5. 24, 42, 72, 54, 36, 60, 12, 66, 30, 18, 48 ÷ 6
6. 50, 10, 20, 30, 40, 60, 45, 55, 35, 15, 25 ÷ 5
7. 44, 28, 20, 48, 12, 32, 40, 36, 16, 24, 8 ÷ 4
8. 27, 18, 6, 36, 24, 9, 33, 21, 12, 30, 15 ÷ 3

B

Practise these exercises in two ways:

(a) State quotient and remainder.
(b) Give answers, thus, 20 ÷ 3 = 3 times 6 + 2, or
    20 = 3 sixes + 2.

Answer rapidly:

1. 15, 13, 7, 11, 5, 21, 9, 3, 17, 19, 15 ÷ 2
2. 17, 7, 11, 19, 8, 14, 23, 13, 5, 16, 20 ÷ 3
3. 14, 19, 9, 27, 31, 15, 22, 25, 38, 17, 39 ÷ 4
4. 22, 16, 38, 44, 19, 7, 33, 27, 39, 48, 13 ÷ 5
5. 23, 11, 37, 45, 20, 7, 34, 52, 29, 17, 57 ÷ 6
6. 17, 32, 54, 25, 13, 68, 41, 29, 39, 58, 61 ÷ 7
7. 39, 58, 19, 49, 28, 62, 13, 44, 69, 37, 23 ÷ 8
8. 119, 117, 113, 17, 86, 118, 90, 109, 91 ÷ 12
9. 65, 97, 45, 110, 78, 56, 32, 75, 23, 85, 98 ÷ 12
10. 107, 68, 73, 95, 25, 59, 64, 28, 35, 102, 80 ÷ 11
11. 20, 50, 40, 70, 100, 60, 30, 90, 85, 63, 79 ÷ 11
12. 89, 58, 26, 67, 39, 76, 16, 32, 48, 83, 60 ÷ 9
13. 15, 34, 55, 70, 80, 20, 42, 64, 86, 82, 75 ÷ 9

SHORT DIVISION

In the written work for division one of two forms may be used. When the divisor is less than 12, there is generally used the form which is called **short division.** In other cases **long division** is employed.

Example 1: Divide 45 dollars by 7. The quotient is 6 dollars and the remainder 3 dollars, and the work is set down thus:

Divisor)Dividend   7)45 dollars
    Quotient and Remainder $6\frac{3}{7}$ dollars

Here the problem to be answered is: What sum taken 7 times will give 45 dollars?

The remainder, 3, is written with the quotient, but with the divisor under it to show that it has not been divided.

Example 2: Divide 23 hundreds by 5. Here the work is set down thus:

5)23 hundreds
$4\frac{3}{5}$ hundreds

Example 3: Divide 36 tens by 5. The work is set down thus:

5)36 tens
$7\frac{1}{5}$ tens

Example 4: Divide 15 units by 5. The work is set down thus:

5)15 units
3 units

Example 5: Divide 2365 by 5. The work is set down thus:

5)2365
473

Explanation: There are four problems: (*a*) What is the number which has to be taken 5 times to give 2 thousand? It is clear that *no* thousands are repeated 5 times. Hence the 2 thousands and the 3 hundreds are taken together. Then comes problem (*b*): What number taken 5 times will give 23 hundred? The quotient is 4 hundred, and the remainder is 3 hundred. The quotient, 4, is written in the *hundreds'* place, and the remainder, 3 hundred, is taken with the 6 tens. Then comes problem (*c*): What number taken 5 times will give 36 tens? The quotient is 7 tens, and the remainder is 1 ten. The quotient, 7, is placed in the *tens'* place, and the remainder, 1 ten, is taken with the 5 units. Then comes problem (*d*): What number taken 5 times will give 15 units? The quotient is 3 units with no remainder. The 3 is placed in the *units'* place, and the complete quotient is 4 hundred + 7 tens + 3 = 473.

Here 2365 = 473 × 5, and, therefore, also = 5 × 473. So there is found the number which has to be taken *5 times* to give 2365 and also the number of *5's* in 2365.

Example 6: 4739 ÷ 8

Work thus: 8)4739
$\overline{592\tfrac{3}{8}}$

Multiply 592 by 8 and add the remainder. How may the work in division be checked?

Example 7: How many pints of milk at 7 cents a pint can be bought for 525 cents? In this the problem is to find what sum taken 7 times will give 525 cents, or to divide 525 cents by 7.

Work thus: 7)525 cents
$\overline{75\text{ cents}}$

Now 525 cents = 75 cents × 7 and is, therefore, also equal to 7 cents × 75. That is, for 525 cents there can be bought 75 pints of milk at 7 cents a pint.

EXERCISE 14

Divide and prove:

1. 2)864	11. 9)718	22. 7)111111
2. 3)981	12. 7)623	23. 6)43085
3. 9)648	13. 12)7000	24. 8)532081
4. 8)1238	14. 4)916	25. 9)487050
5. 9)369	15. 3)7070	26. 12)82695
6. 11)858	16. 2)5007	27. 10)46250
7. 7)504	17. 11)57189	28. 10)983600
8. 6)3255	18. 9)102348	29. 10)5213
9. 4)2367	19. 12)459684	30. 10)67368
10. 3)4163	20. 8)300024	31. 10)729364
	21. 5)32786	32. 10)34020

33. Without dividing, write the quotients:

      (a) 15640; 72900; 8205; 4793; 6452 ÷ 10.

      (b) 1165; 22222; 90030; 8000; 76008 ÷ 10.

34. Divide 4780 by 10 and then divide the quotient by 2. Now multiply the *second* quotient by 20.

35. Divide 7350 by 10 and then divide the quotient by 3. Now multiply the second quotient by 30.

36. Divide 8340 by 10 and the quotient by 6. Now multiply the last quotient by 60.

37. Divide 4750 by 50.

38. Divide 6439 by 70. The work is set down thus:

$$70)\overline{6439}$$
$$91\tfrac{69}{70}. \quad \text{Prove.}$$

39. In the following, set down the work as shown in Example (38) and prove answers:

$$4368 \div 90; \quad 25820 \div 60; \quad 53729 \div 110;$$
$$8453 \div 120; \quad 21486 \div 80.$$

40. Divide each of these numbers by 100 and prove answers:

$$52600; \quad 37500; \quad 84300; \quad 15700.$$

41. Divide each of these numbers by 100 and prove answers:

$$5368; \quad 72596; \quad 13704; \quad 21005.$$

42. Divide each of the following by 200, then by 500, and then by 700:

$$82600; \quad 47500; \quad 84700; \quad 15976; \quad 24003.$$

LONG DIVISION

ORAL EXERCISE

1. Divide 36 by 4 and also by 6. Which gives the larger quotient?

2. Divide 90 by 5 and also by 9. Which gives the smaller quotient?

3. Divide 96 by 6 and by 8. Which gives the larger quotient?

4. If the same dividend is divided by 3 and by 5, which would give the larger quotient?

5. If the same dividend is divided by 3, by 4, and by 5, which would give the larger quotient, 3 or 5? 4 or 5? 3 or 4?

6. Test Example (5) by using 240 as the dividend.

7. If the same dividend is divided by 6, by 7, and by 8, which quotient would be the largest? which the smallest?

8. The same dividend is divided by 6, by 9, by 4, by 10, by 3, by 7. Which will give the largest quotient? the next largest? the next largest? and so on.

9. A pupil found that a certain dividend when divided by 20 gives a quotient of 45, and when divided by 30 a quotient of 30. If this dividend is divided by 27, between what numbers will the quotient lie? Why? To which number will it be the nearer? Why?

10. If 600 is to be divided by 28, name two numbers between which the quotient will lie. To which of these numbers will the quotient be the nearer?

11. If 840 is divided by 32, name two numbers between which the quotient will lie. To which of the numbers will the quotient be the nearer?

12. For each quotient name two numbers between which it must lie when: $480 \div 37$; $200 \div 43$; $630 \div 76$; $420 \div 24$; $547 \div 65$.

From the foregoing Exercise it is evident that if 14245 is to be divided by 37, the quotient will be greater than that which is obtained by dividing the dividend by 40 and less than that obtained by dividing by 30, but it will be *nearer* the first of these quotients. For this reason *40* may be used as a **trial divisor** to aid in finding the quotient, when the *real* divisor is 37.

Example 1: Divide 14245 by 37. The work is set down thus:

$$
\begin{array}{r}
385 \\
37)\overline{14245} \\
111 \\
\hline
314 \\
296 \\
\hline
185 \\
185 \\
\hline
\end{array}
$$

Explanation: It is seen that 142 hundred is the first part of the dividend which can be divided by 37. The problem then is: What number must be taken 37 times to

give 142 hundred? Using the *trial* divisor, 40, instead
of 37, the problem is changed to: What number must be
taken *40 times* to give 142 hundred? This is found by
dividing 142 hundred by 40, or 14 hundred by 4, the
quotient for which is 3 hundred. Placing the 3 over the
hundreds' place, multiplying 37 by 3, and subtracting the
product 111 hundred from 142 hundred, there remains
31 hundred. This taken with the 4 tens gives 314 tens
to be next divided by 37. Here again using the 40 as
trial divisor instead of 37, the problem is: What number
must be taken 40 times to give 314 tens? This is found
by dividing 314 tens by 40. The quotient is 7 tens.
Placing the 7 tens over the tens' place, multiplying 37
by 7, and subtracting the product, 259 tens, from 314 tens,
the remainder is found to be 45 tens. This *remainder* is
too large, being greater than 37, hence the *quotient figure*,
7, is *too small*. Erase it and try 8. Multiplying and
subtracting as before, the remainder is now found to be
18 tens. This with the 5 units gives 185 units to be
divided by 37. Using 40 as a trial divisor, the quotient
is found to be 4 units. Placing the 4 over the units'
place and multiplying and subtracting as before, the re-
mainder is found to be 37, which is again too large. So
the quotient figure, 4, is too small. Erase it and try 5
and proceed as before. This will be found to be the
correct quotient figure, and the whole quotient is now
385. That is—

$$14245 = 385 \times 37 = 37 \times 385. \qquad \text{Prove.}$$

Practice in the use of trial divisors will in time lessen the
number of *unsuccessful* trials.

Example 2: Divide 27589 by 64. Here 60 may be
used as the trial divisor, and the work will be set down
thus:

$$431\tfrac{5}{64}$$
$$64)\overline{27589}$$
$$256$$
$$\overline{}$$
$$198$$
$$192$$
$$\overline{}$$
$$69$$
$$64$$
$$\overline{}$$
$$5$$

That is $27589 = (431 \times 64) + 5$
or $27589 = (64 \times 431) + 5$.

Show that the quotients in the above Examples can be obtained quite easily by using 30 for the trial divisor in the first and 70 in the second.

It is evident that there will always be a choice between two trial divisors. Experience has shown that time will be saved by knowing how to make the choice without hesitation. For this reason the smaller of the two possible trial divisors is the one generally chosen. The trial quotient figure, if not correct, will in that case always be diminished.

EXERCISE 15

Divide, and prove your work:

1. 24384 by 48
2. 16686 by 54
3. 32199 by 57
4. 680096 by 72

5. 430765 by 68
6. 568374 by 59
7. 378096 by 88

8. 24329 by 21; by 31; by 61; by 81
9. 30504 by 42; by 52; by 62; by 72
10. 379830 by 15; by 25; by 75; by 95
11. 593004 by 59; by 79; by 83
12. 100000 by 56; by 68; by 77.

SECTION II

CHECKS IN ADDITION

1. Each column is added up and down. Compare results.

2. Add each column separately, then total the sums of all the columns.

The following Example will show the method:

$$\begin{array}{r} \$\ 25\cdot78 \\ 39\cdot63 \\ 48\cdot75 \\ 62\cdot37 \\ \hline \$176\cdot53 \end{array}$$

Adding each column separately, we have these totals:

$$\begin{array}{r} 23 \\ 23 \\ 24 \\ 15 \\ \hline \$176\cdot53 \end{array}$$

The sum of the separate totals is $176·53.

EXERCISE 16

1. Add and test these four questions in 4 minutes:

(a)	(b)	(c)	(d)
$475·32	$763·42	$598·64	$723·54
69·58	895·85	276·38	895·37
367·42	763·29	493·65	28·69
9·87	483·65	728·37	5·43
425·34	78·87	89·42	986·59
78·49	946·58	8·69	473·82

2. Add and test the following four questions in 6 minutes:

(a) $987·65	(b) $786·39	(c) $985·43	(d) $467·81
483·78	594·87	276·98	293·84
695·99	648·32	15·43	684·79
468·37	780·96	879·68	573·68
825·96	478·30	8·57	264·35
748·35	64·82	694·83	587·49
65·84	7·54	725·78	693·85
738·59	896·47	946·59	468·73

3. Add and test the following questions in 8 minutes:

(a) $5837·29	(b) $2938·67	(c) $5946·38
2764·83	5286·38	2759·47
9407·08	9895·49	4863·89
6835·47	3964·73	5987·65
9684·89	8275·89	8743·29
5643·75	4763·34	9384·64
9859·48	2894·75	7865·78
6743·72	8647·49	8939·64
9834·56	7835·63	7654·83

4. Add and test each question in 2 minutes:

(a)$14962·08	(b) $363·97	(c) $493·64
27589·73	28·54	280·09
463·29	758·92	59·60
8758·46	983·47	827·54
98906·53	655·68	694·73
79483·75	753·47	928·59
84569·48	9·89	476·68
	948·65	829·54
	753·84	673·29
	296·53	486·58

CHECKS IN SUBTRACTION

To test the correctness of an answer in subtraction, add the remainder (answer) to the subtrahend. The sum should equal the minuend.

EXERCISE 17

1. Subtract and test two questions in 1 minute:

(a) $5894·26	(b) $9473·72	(c) $5475·06	(d) $4000·00
2938·78	4829·37	2999·39	2756·43

(e) $7948·63	(f) $7478·00	(g) $2895·42	(h) $3281·63
1299·99	1297·38	1289·76	1899·47

2. Find each balance after the following deposits and withdrawals have been made. Check your last balance in each Example by adding withdrawals and subtracting the sum from the deposit.

(a) Deposit $100. Withdrawals, $25·37, $16·09, $32·33, $15·06.

(b) Deposit $1000. Withdrawals, $246·33, $347·20, $199·39.

(c) Deposit $8000. Withdrawals, $3415·03, $1699·48, $2543·06.

ORAL EXERCISE IN THE MAKING OF CHANGE

Make the change in the following transactions orally and by the additive method:

Amount of Purchase		Amount given
1. $4·21		$5·00
2. $1·18		$5·00
3. ·87		$2·00
4. $1·31	cheque	$2·50
5. $1·56	cheque	$1·75
6. $3·42	cheque	$4·00
7. $3·99		$5·00
8. $4·18		$5·00
9. $5·03		$6·00
10. $5·17		$6·00
11. $6·23		$10·00
12. $8·19		$10·00

CHECKS IN MULTIPLICATION

1. Interchange multiplier and multiplicand and find the product. This product should be the same as the former product.

2. Divide the product by (*a*) the multiplier, or (*b*) the multiplicand. If the quotient obtained in (*a*) is the multiplicand or in (*b*) the multiplier, the product is correct.

EXERCISE 18

1. Find the value of and test one example in 2 minutes:

(*a*) 8597 × 418	(*f*) 8659 × 597
(*b*) 6408 × 593	(*g*) 6742 × 681
(*c*) 8749 × 426	(*h*) 5093 × 576
(*d*) 7054 × 369	(*i*) 3020 × 498
(*e*) 9824 × 438	(*j*) 5486 × 307

ORAL EXERCISE

Find the value of:

1. 8× 5×7	**8.** 9× 8× 5	**15.** 9× 5×10
2. 8× 6×3	**9.** 6× 5×12	**16.** 7× 6× 8
3. 7× 9×5	**10.** 7× 9× 6	**17.** 8× 8× 8
4. 11×10×9	**11.** 11×12×10	**18.** 9× 9× 9
5. 8× 6×7	**12.** 9×10×12	**19.** 11×11×11
6. 12× 5×8	**13.** 7× 4× 9	**20.** 12×12×12
7. 4× 9×8	**14.** 8× 6× 7	

Market Quotations, November 13th, 1919

1. *Produce:*

Eggs—New laid, 78c. per doz.
      No. 1   61c. " "
      Select   65c. " "
Butter—Creamery Prints   64c. a lb.
        Choice Dairy Prints 55c. "
Oleomargarine—choice   37c. "
Cheese   31c. "

4

2. *Poultry:*

Spring chickens—(dressed)   28c. a lb.
Hens—   "   26c.   "
Ducklings—   "   28c.   "
Turkeys—   "   37c.   "

8. *Grain, Flour and Feed:*

Ontario Fall wheat No. 1 $2·06 per bu.
  "   Spring wheat No. 1 $2·08 "   "
  "   oats   ·95   "   "
  "   barley $1·65 "   "
  "   peas $2·50 "   "
Shorts $52·00 per ton
Bran $45·00 "   "
Feed flour $3·50 per bag
Flour $9·60 per cwt.

4. *Fruit:*

Valencia oranges $6·50 per case
California lemons $7·25 "   "
Florida grape fruit $4·75 "   "
Grapes $4·50 " box
Apples (Spy) $7·50 " bbl.

5. *Vegetables:*

Tomatoes   ·75 per basket
Potatoes $2·35 " bag
Carrots $1·25 "   "
Celery   ·75 a doz.

EXERCISE 19

Using prices quoted above, find the value of the following purchases:

1. 4 doz. No. 1 eggs
   5 lb. of dairy butter
   1 spring chicken weighing 4 lb.
   2 bags of potatoes.

2. 5 lb. of margarine
   2 lb. of cheese
   1 turkey weighing 11 lb.
   2 baskets of tomatoes
   1 cwt. of flour.

3. 47 bu. of Spring wheat
   75 "   " oats
   32 "   " barley.

4. 3 cases of oranges
   2 cases of lemons
   4 bbl. of apples.

5. 15 doz. new laid eggs
   12 lb. of creamery butter
   2 ducklings together weighing 11 lb.
   1 bag of carrots.

6. 3 tons of bran
   4 tons of shorts
   7 bags of feed flour
   1 cwt. of flour.

7. 5 doz. celery
   4 bags of potatoes
   3 bbl. of apples
   2 bu. of peas.

8. 4 dressed chickens, total weight 18 lb.
   3   "   hens   "   "   17 "
   6   "   turkeys   "   "   75 "
   7   "   ducklings "   "   49 "

CHECKS IN DIVISION

1. The remainder added to the product of the quotient and the divisor should equal the dividend.

2. Take the remainder from the dividend, divide this difference by the quotient, and the answer should be the first divisor.

EXERCISE 20

Divide and test one question in 2 minutes:

(a) 46783 by 267   (f) 48756 by 354
(b) 74365 by 265   (g) 68473 by 378
(c) 94284 by 463   (h) 59506 by 809
(d) 82967 by 349   (i) 40738 by 607
(e) 93825 by 429   (j) 59476 by 833

EXERCISE 21

(Practical Problems in Addition, Subtraction, Multiplication, and Division)

1. Harry deposited in the Savings Bank $4·75 in January, $8·63 in February, $5·91 in March, $7·42 in April, $3·78 in May, and $4·75 in June. How much did he deposit in the six months?

2. Mary went to the store with a $20-bill. She bought 9 yards of cloth for a dress at $1·35 a yard. What change should she receive?

3. Find the cost of 13 tons of coal at $11·50 a ton.

4. A farmer makes 78 gallons of maple syrup. What should he receive for it at $1·75 a gallon?

5. How many lb. of butter at 49c. a lb. can I purchase with $31·85?

6. In a bu. of wheat there are 60 lb. How many bushels and pounds are there in a load which weighed 3498 lb.?

7. What is the value of 18 bags of potatoes if each bag contains 2 bushels and a bushel is worth $1·25?

8. What is the total amount of the following purchases: 2 lb. of butter at 47c. a lb., 6 lb. of fish at 18c. a lb., 3 bars of soap at 12c., 4 spools of thread at 7c. a spool?

9. In a bu. of oats there are 34 lb. How many bushels and pounds are there in a load which weighs 3498 lb.?

10. If a man earns $43·75 in 7 days, what should he earn in 21 days?

11. How many tons of hay at $14 per ton must I sell to pay for 200 bushels of wheat worth $2·17 a bushel?

12. If oranges are worth 48c. a doz., what are 8 boxes worth, each box containing 30 doz.?

13. A farmer sold a farm for $7950. It cost him $6527. Find his gain.

NUMBERS, FACTORS, AND DIVISORS

ODD AND EVEN NUMBERS

From the following list of numbers select those into which 2 will divide and leave no remainder: 7, 8, 11, 14, 18, 21, 24, 29. These numbers are called *even* numbers. The others—7, 11, 21, 29, each of which leaves a remainder when 2 is divided into it, are called *odd* numbers.

FACTORS

What is the product of 3 and 5?

3 and 5 are said to be the **factors** of 15.

Of what number are 4 and 7 the factors? 3 and 8? 6 and 11? 4 and 9? 5 and 12?

Give the factors of the following numbers:

63, 84, 72, 96, 75, 88, 108, 125.

INTEGERS

Whole numbers such as 4, 5, 7, 17, 22, 98, etc., are called **integers.**

Name six numbers which are integers. Name a number which is not an integer.

DIVISOR AND FACTOR

Any integer which will divide another integer without leaving a remainder is an **exact divisor** of that integer.

Name at least two exact divisors of each of the following numbers:

12, 15, 16, 27, 33, 42, 45, 56, 64, 72.

The exact divisors which multiplied together make an integer are called *the factors* of that integer.

An exact divisor of an integer is always a factor of that integer.

Name a factor of: 10, 22, 28, 32, 45, 77, 81, 90.

Name the factors of: 10, 21, 28, 42, 56, 63, 72, 88.

COMMON FACTOR

The factors of 35 are 5 and 7.

The factors of 45 are 5 and 9.

What factor is common to both 35 and 45?

Five is said to be a **common factor** of 35 and 45.

ORAL EXERCISE

1. Select the even numbers from the following list of numbers: 9, 14, 38, 46, 53, 74, 82, 85.

2. Select the odd numbers from the following list: 12, 17, 34, 37, 62, 65, 69, 72, 79, 81, 84.

3. State the factors of: 48, 54, 72, 65, 110, 108, 120, 144, 240.

4. What common factors have: 32 and 48; 25 and 40; 33, 55, 88; 96, 108, and 120; 36, 45, and 63; 24, 56, 88; 110, 121, and 132?

5. Factor 32, 48, 56, 84, 96, in as many ways as you can.

PRIME NUMBER AND PRIME FACTOR, COMPOSITE NUMBER

What factors have the numbers: 5, 7, 11, 17, 23, 29, 1, 47?

A number which has no factors except the number 1 and itself is called a **prime number.**

A prime number which is a factor of a number is called a **prime factor.**

Name six prime numbers.

Select a prime factor of each of the numbers: 6, 8, 12, 14, 20, 24, 36, 42, 45, 48, 54, 56.

A number, such as 12, 15, 18, 36, or 44, composed of factors other than itself and the number 1 is called a **composite number.**

Which of the following numbers are prime and which are composite: 3, 6, 13, 23, 24, 32, 37, 41, 45, 49, 59?

TO EXPRESS A COMPOSITE NUMBER AS THE PRODUCT OF ITS PRIME FACTORS

Sometimes it is necessary to express a number which is composite as the product of its prime factors. The following Example will show how this is done:

$$924 = 2 \times 462$$
$$= 2 \times 2 \times 231$$
$$= 2 \times 2 \times 7 \times 33$$
$$= 2 \times 2 \times 7 \times 3 \times 11.$$

Explanation:

The factors of 924 are 2 and 462; 2 is a prime factor, while 462 is composite; a prime factor of 462 is 2; $924 = 2 \times 2 \times 231$. The number 231 is composite. 7 is a prime factor of 231. Hence, the number $924 = 2 \times 2 \times 7 \times 33$, and the prime factors of 33 are 3 and 11.

Hence, $924 = 2 \times 2 \times 7 \times 3 \times 11$.

EXERCISE 22

1. Express each of the following numbers as the product of its prime factors:

(a) 324	(d) 392	(g) 605	(j) 1962
(b) 336	(e) 420	(h) 840	(k) 4335
(c) 390	(f) 495	(i) 882	(l) 5390

DIVISION BY FACTORS

If a divisor is composite and can easily be resolved into factors, these factors, instead of the divisor itself, may be used to make the division. The following Example will illustrate the method:

Divide 4284 articles into groups of 84.

1. Dividing in the usual way:

$$
\begin{array}{r}
51 \\
84)\overline{4284} \\
420 \\
\hline
84 \\
84 \\
\hline
\end{array}
$$

There are 51 groups
with 84 in each group.

2. Dividing by factors:

The factors of 84 are 7 and 12.

$$
\begin{array}{l}
7)\overline{4284} \text{ articles} \\
12)\overline{612} \text{ groups of } 7. \\
\overline{51} \quad \text{ '' } \quad \text{ '' } 84.
\end{array}
$$

Explanation:

The problem is to divide 4284 articles into groups of 84. The 4284 articles are first divided into groups of 7. There are 612 of these. To make the groups of 84, it is necessary to put together 12 groups of 7 as often as possible out of the 612 groups of 7. Twelve is contained in 612, 51 times. Hence, there are 51 groups of 84.

EXERCISE 23

Divide, using two factors—Examples (1–9); using 3 factors—Examples (10–12):

1. 1305 by 15	5. 5754 by 42	9. 19096 by 56
2. 8631 by 63	6. 12042 by 54	10. 3720 by 30
3. 4257 by 33	7. 40032 by 48	11. 96915 by 105
4. 6264 by 72	8. 30912 by 84	12. 91960 by 110

BILLS

Mrs. Wm. Stone of 500 Grace Street, Toronto, went to Mr. F. Manning's grocery store, 910 West Street, Toronto, on Oct. 21st, 1919, to make a purchase. Mr. Manning wrote in his order book a list of the groceries which Mrs. Stone wished to buy, and then filled the order. Suppose Mrs. Stone bought the following gro-

ceries and paid cash for them: 3 doz. eggs @ 65c. a doz.,
1 lb. of cheese @ 39c. a lb., 2 lb. of butter @ 57c. a lb., 1 lb.
of tea @ 50c. a lb.   She received from Mr. Manning along
with the goods a **bill** of the goods which she bought.
This was the form of the bill which she received.   Mr.
Manning kept a copy of it.   Why?

	Toronto, Oct. 21st, 1919.			
Mrs. Wm. Stone,				
500 Grace Street, Toronto.				
Bought of	F. MANNING			
	910 West Street, Toronto.			
	Dealer in			
	PROVISIONS, GROCERIES, and FRUITS			
1.	3 doz. eggs	65c.	$1	95
2.	1 lb. cheese	39c.		39
3.	2 lb. butter	57c.	1	14
4.	1 lb. tea	50c.		50
			3	98
	(Paid)			
	F. M.			

This bill is footed (totalled) and receipted.

If Mrs. Stone had not paid for the goods, the word
"Charge" would have taken the place of "Paid," and
Mr. Manning would then have entered the items in
Mrs. Stone's account.

EXERCISE 24

1. Make out in proper form the following bills.   In
each Example suppose one of your class-mates is the
salesman and you the buyer.   Receipt the bill in each case.

(a) 1 Reader 15c.
     1 P. S. Arithmetic 10c.
     pencils 5c.
     2 work books @ 5c.
     1 pen and penholder 7c.
     1 bottle ink 10c.

(b) 4 lb. ham @ 65c.
     3 lb. butter @ 55c.
     1 can tomatoes 25c.
     1 pk. potatoes 60c.

(c) 2 packages breakfast food @ 15c.
     2 lb. cheese @ 37c.
     3 lb. fish @ 18c.
     1 bottle catsup 30c.
     1 can corn 25c.

(d) 1 fishing-tackle $1·90
     1 doz. hooks @ 30c. a doz.
     1 fish-line @ $1·25
     1 basket $1·75
     1 fishing-pole $2·50

(e) 2 suits of clothes @ $27
     2 hats @ $3·25
     3 ties @ $1·25
     4 collars @ 25c.
     6 handkerchiefs @ 20c.

2. Make out in proper form a bill such as might be made out by the bookkeeper of a hardware store. Make the bill contain 4 items. Foot and receipt the bill.

3. Make out in proper form a bill such as might be made out by the bookkeeper of a fruit store. Make the bill contain 5 items. Charge the bill of goods.

4. Make out in proper form the bills that the book-keepers of the following stores might make out: A cloth-

ing store, a furniture store, a meat store, a drug store, a stationery store, a dry-goods store. Make each bill contain 6 items and receipt it.

ACCOUNTS

It has already been shown that in a *bill* there are given the details of a business transaction between two persons, which has taken place on a certain date. In an **account** there is usually a record of a number of transactions between the same persons, which have taken place on different dates during a period of time, say a month.

The following Example will explain the meaning of an *account* and illustrate the difference between a bill and an account.

Suppose Mrs. Stone buys from Mr. Manning on different days the following groceries and asks him to charge each purchase to her account:

1919, Nov. 3rd.	3 packages breakfast food @ 15c.
	3 lb. oatmeal @ 10c.
Nov. 8th.	3 lb. ham @ 68c.
	2 cans tomatoes @ 25c.
	5 lb. butter @ 60c.
Nov. 15th.	2 doz. eggs @ 65c.
	1 lb. cheese @ 40c.
	3 lb. biscuits @ 30c.

At the time each purchase is made, Mr. Manning makes out a bill of the goods purchased and gives it with the goods to Mrs. Stone. He also keeps a copy of each bill and enters the items of it in Mrs. Stone's account. If on Nov. 20th. she pays $3 on her account and on Nov. 28th. she pays $2, the following is the form of the account which Mr. Manning would send her on Dec. 1st.:

		910 West Street, Toronto, Ont., Dec. 1st, 1919.				

Mrs. Wm. Stone,
    500 Grace Street, Toronto.
       In Account with

F. MANNING

Dealer in

PROVISIONS, GROCERIES, and FRUITS

		Dr.			
Nov.	3	To 3 packages breakfast food @ 15c.		45	
		" 3 lb. oatmeal @ 10c.		30	
Nov.	8	" 3 lb. ham @ 68c.	2	04	
		" 2 cans tomatoes @ 25c.		50	
		" 5 lb. butter @ 60c.	3	00	
Nov.	15	" 2 doz. eggs @ 65c.	1	30	
		" 1 lb. cheese @ 40c.		40	
		" 3 lb. biscuits @ 30c.		90	8 89
		Cr.			
Nov.	20	By Cash	3	00	
Nov.	28	" "	2	00	5 00
		Balance due F. Manning			3 89

EXERCISE 25

Make out the following accounts in good form. In each Example suppose you are the merchant and one of your class-mates the buyer.

1. April 5th.   3 yd. ribbon @ 60c.
                  4 yd. dress goods @ $1·50
   April 12th.   1 hat $6·50
                  1 coat $27·50
   April 18th.   1 pair boots $14·00
                  1 pair rubbers $1·75

On April 28th, $25 was paid on account. Account rendered April 30th.

2. Jan. 3rd.	1 pair skates $5·50
	1 pair boots $4·50
Jan. 9th.	1 hockey stick 75c.
	1 puck 75c.
Jan. 15th.	1 breast pad $7·75
	1 pair gloves $3·50

On Jan. 25th, $10 is paid. On Jan. 30th, $5 is paid. Account rendered Jan. 31st.

3. Feb. 2nd.	1 suit of clothes $38·00
	1 overcoat	32·00
	1 cap	2·25
	1 pair gloves	1·75
Mar. 15th.	1 tie	1·75
	1 neck scarf	2·50

On March 30th, paid $25. On April 20th, paid $25. Account rendered April 30th.

4. May 1st.	3 lb. pork @ 40c.
	1 chicken 4 lb. @ 26c.
May 30th.	2 lb. beefsteak @ 35c.
	2 lb. cold tongue @ 65c.
June 15th.	5 lb. roast @ 42c.
	2 lb. tenderloin 52c.

On June 30th, account is paid in full. Render account receipted.

RECEIPTS

John Hill pays James Burns $37·50 for work done and asks Mr. Burns for a receipt. The following is the form which the receipt might take:

Kingston, Oct. 15th, 1919.

$37·50

Received from......John Hill..............
Thirty-seven................50/100 Dollars
for work done.

                              JAMES BURNS

EXERCISE 26

1. What kind of receipt does a postman give when he delivers a registered letter?

2. Write the receipt a grocer would give in payment of a monthly account of $47·75.

3. James Smith receives from Henry Brown $125 for 5 months' rent. Write the receipt.

4. Arthur Mann pays his coal bill—$137·50—to James Squire, the coal-dealer. Write the receipt.

5. Samuel Perry pays to Amos Farrell $150 balance due on his automobile. Write the receipt.

6. Assuming that you are a collector for a newspaper, make out a receipt to a subscriber who has paid you $4·00 for a year's subscription to a Toronto daily paper.

7. Write out a receipt which one might receive from a family doctor for medical services rendered.

EXERCISE 27

(Aggregates and Averages)

1. Find the sum of the 5 numbers, 6, 7, 8, 9, 10.

2. What number taken 5 times is equal to that sum? 8 is said to be the **average** of the 5 numbers, and 40 is said to be the sum, or **aggregate,** of the 5 numbers.

3. Find the average of the 4 numbers, 7, 11, 15, and 19.

4. Find the aggregate and the average of the following weights: 38 lb., 54 lb., and 73 lb.

5. The weights of 4 boys are: 78 lb., 84 lb., 69 lb., and 92 lb. What is the aggregate and average weight of the 4 boys?

6. In six working days of a week Mary earned the following amounts: 75c., $1·10, 85c., 60c., 95c., and 55c. How much did she earn altogether? What is the average daily amount which she earned?

7. Five boys earned by the sale of vegetables from their school gardens the following amounts: $6·32, $4·61, $3·75, $5·02, and $2·60. What was the aggregate amount from the five gardens? What was the average amount obtained from each garden?

8. Grace put in the Penny Savings Bank in 4 weeks the following sums: $2·57, $3·62, $1·89, and $2·24. What was the average sum put in each week?

9. Harry purchased 7 school books and paid as follows for them: 15c., 25c., 18c., 9c., 14c., 10c., and 7c. What was the aggregate and average price of the books?

10. A man delivered 4 loads of coal. The weight of the first load was 2150 lb., of the second 1975 lb., of the third 2260 lb., and of the fourth 2315 lb. What was (a) the aggregate weight of the 4 loads? (b) the average weight of the loads?

LIQUID MEASURE

Find out, by using the pint, quart, and gallon measures, and some water, the number of pints in one quart and the number of quarts in one gallon.

Fill in the blanks in the following table:

.... pints = 1 quart
.... quarts = 1 gallon.

EXERCISE 28

1. A pail of milk contains 2 gal. How many quarts does it contain? How many pints? What is the value of the pail of milk at 7 cents a pint?

2. How many pints are there in 10 quarts? in 12 quarts? in 9 quarts? in 15 quarts? in 25 quarts?

3. What is the value of a 10-quart pail of milk at 6 cents a pint?

4. How many pint bottles can a milkman fill from a 5-gal. can of milk?

5. A grocer buys a barrel of vinegar which contains 31½ gal. What does it cost him at 8c. a quart?

6. The gasoline tank in an automobile holds 8 gal. What will it cost to fill the tank with gasoline at 42c. a gal.?

7. What is a bbl. of coal-oil (42 gal.) worth at 24c. a gal.?

8. What is the value of a 3-gal. can of maple syrup at 70c. a quart?

9. Three casks of wine hold, respectively, 40 gal., 35 gal., and 27 gal. How many quart bottles can be filled from the three casks?

DRY MEASURE

Name some dry articles purchased by the pint, by the quart, by the gallon, by the peck, by the bushel.

Use sawdust or grain, and the pint, quart, gallon, peck, and bushel measures to find the number of pints in a quart, the number of quarts in one gallon, the number of gallons in one peck, and the number of pecks in one bushel.

Fill in the following blanks, then memorize the table:

> .... pints = 1 quart (qt.)
> .... quarts = 1 gallon (gal.)
> .... gallons = 1 peck (pk.)
> .... pecks = 1 bushel (bu.).

1. How many pints in 6 qt? in 9 qt.? in 11 qt. 1 pt.?

2. How many quarts in 22 pt.? in 32 pt.? in 46 pt.?

3. What is the value of 5 qt. of currants at 12c. a pt.?

4. What is the value of 33 pints of cherries at 8c. a qt.?

5. A basket contained 15 qt. of raspberries. What are the berries worth at 15c. a pt.?

6. How many quart boxes can be filled from 4 baskets of strawberries, each basket containing 16 quarts? What is their value at 30c. a box?

7. How many gal. in 2 pk.? in 3 pk.? in 4 pk. 1 gal.? in 5 pk. 1 gal.?

8. How many pk. and gal. in 10 gal.? in 14 gal.? in 22 gal.? in 25 gal.? in 31 gal.?

9. A farmer feeds a team of horses 3 pk. of oats each day. What is the cost of feeding the team for 1 week if oats are worth 10c. a gal.?

10. What is the value of 3 pk. 1 gal. of oats at 24c. a pk.?

11. How many pk. in 3 bu.? in 7 bu.? in 12 bu.? in 6 bu. 2 pk.? in 9 bu. 3 pk.?

12. How many bu. and pk. in 8 pk.? in 12 pk.? in 20 pk.? in 33 pk.? in 46 pk.?

13. What is the value of a bag of potatoes, $1\frac{1}{2}$ bu., @ 40c. a pk.?

14. A grocer purchases 3 bu. of plums. What are the plums worth at 6c. a qt.?

CANADIAN MONEY

1. Ask your father the rate of his taxes for last year, also for this year. What is his answer?

Canada has no coin the value of which is 1 mill. But it has a unit called a mill; 10 of these units are equal in value to 1 cent.

Fill in the following blanks and then memorize the table:

    .... mills = 1 cent (c.)
    .... cents = 1 dollar ($)
    One half of 1 dollar = ...cents = ...quarters of 1 dollar
    One quarter of 1 dollar = ...cents
    Three quarters of 1 dollar = ... cents.

ORAL EXERCISE

1. How many mills are there in 1 cent? in 2 cents? in 3 cents? in 5 cents? in 10 cents?

2. How many cents are there in 20 mills? in 40 mills? in 60 mills? in 100 mills? in 1000 mills?

3. How many mills in $2? in $3? in $11? in $20?

4. How many dollars are there in 1000 mills? in 5000 mills?

5. What are the taxes on a house worth $3000 if the taxes are 30 mills on every dollar?

6. How many dollars and cents are there in each of the following number of mills: 4500 mills? 4560 mills? 5790 mills? 9260 mills? 15460 mills?

7. Find the sum of the following in dollars and cents: $4, 37c., $¾, 700 mills, $9·28.

BRITISH STERLING MONEY

NOTE TO TEACHER.—Procure, if possible, an English farthing, a penny, a shilling, and a pound sterling.

1. Compare the value of the English penny with a Canadian cent.

2. To what Canadian coin is the English shilling almost equal?

3. What is the value of the English pound in Canadian money?

4. Complete the following table, then memorize it:

.... farthings = 1 penny (d.)
....pence = 1 shilling (s.)
.... shillings = 1 pound (£).

5. Complete the following table:

1 farthing = ...cent (nearly)
1 penny = ... cents (nearly)
1 shilling = ... cents (nearly)
1 pound = ...dollars and ....cents.

EXERCISE 29

1. How many pence in 2 shillings? in 5 shillings? in 20 shillings? in 1 pound?

2. How many shillings and pence in 24 pence? in 36 pence? in 30 pence? in 50 pence? in 100 pence?

3. How many pence in 756s.? in 12s. 3d.? in 15s. 9d.?

4. How many shillings in £1? in £5? in £12? in £25?

5. How many shillings in £5 6s.? £12 7s.? £25 2s.?

6. Reduce £12 5s. 3d. to pence.

7. Reduce 985 pence to £ s. d.

8. Find the value in £ s. d. of 157 yards of ribbon @ 6d. a yard.

LONG, OR LINEAR, MEASURE

Complete the following table and memorize it:

.... inches (in.) = 1 foot (1 ft.)
.... feet (ft.) = 1 yard (1 yd.)
1760 yards (yd.) = 1 mile.

ORAL EXERCISE

1. Express the width of your school-room in feet and inches.

2. Express the length of your school-room in feet and inches.

3. Measure the length and the width of your school-room in yards, feet, and inches.

4. Measure the length and the width of the school grounds in yards and feet. How many feet in the length? How many feet in the width?

5. Measure the width of the road or street in yards. How many feet wide is it?

6. How many inches in 6 ft. 3 in.? in 7 ft. 8 in.? in 9 ft. 5 in.? in 12 ft. 2 in.?

7. How many feet and inches in 27 in.? in 39 in.? in 56 in.? in 75 in.? in 87 in.? in 100 in.?

8. Measure in inches the length of the black-board in the front of the room. How many feet and inches in its length?

9. How many feet in 9 yd.? in 13 yd.? in 17 yd.? in 24 yd.?

10. How many yards and feet in 32 ft.? in 68 ft.? in 76 ft.? in 85 ft.? in 93 ft.?

11. A field is 220 yards long. How many feet long is it?

12. How many yards in 1 mile? in 2 miles?

EXERCISE 30

1. How many yards in 3 miles? in 7 miles? in 4 mi. 220 yd.? in 5 mi. 440 yd.?

2. What part of a mile is 880 yd.? is 440 yd.?

3. If a field is 220 yards wide, how many such fields would there be along a road 1 mile 440 yards long?

4. What will it cost to pave a street 1540 yards long at $3·50 a foot?

5. What will it cost to dig a ditch 1½ miles long at $1·25 a yard?

6. How long will it take a boy to run 100 yards if he runs 20 feet a second?

7. Find how many feet in 1 mile.

8. An aeroplane flies 88 yards a second. How many miles will it fly in 1 minute? in 1 hour?

9. Find the cost of 20 miles of telephone wire at 35c. a lb., if a lb. will stretch 80 feet?

10. An automobile is travelling at the rate of 24 miles an hour. How many yards does it travel in 1 minute?

AVOIRDUPOIS WEIGHT

Name some articles that are bought by the ounce, by the pound, by the hundredweight, by the ton.

Find out by experiment how many ounces there are in 1 pound.

Fill in the blanks of the following table, then memorize it.

> .... ounces = 1 pound (lb.)
> .... pounds = 1 hundredweight (cwt.)
> .... pounds = 1 ton (T.)

EXERCISE 31

1. How many ounces in 1 lb.? in 3 lb.? in 4 lb.? in 6 lb.? in 2½ lb.? in 3¼ lb.?

2. How many pounds and ounces in 32 oz.? 48 oz.? 60 oz.? 84 oz.?

3. How many ounces in 3 lb. 5 oz.? 5 lb. 9 oz.? 7 lb. 4 oz.?

4. How many pounds and ounces in 42 oz.?   69 oz.?
88 oz.?   114 oz.?

5. What is the value of 32 oz. of butter at 65c. a lb.?

6. What is the value of 48 oz. of margarine at 36c.
a lb.?

7. What is the value of 56 oz. of coffee at 50c. a lb.?

8. What is the value of 600 lb. of flour @ $10·50 a
cwt.?

9. What is the value of 800 lb. of coal @ 65c. a cwt.?

10. What is the value of 3 beef cattle, total weight
3950 lb., at $14 a cwt.?

11. A load of hay weighs 3000 lb.   What is its value
at $16 a ton?

12. What is the value of 4000 lb. of bran at $54 a ton?

In business it is generally more convenient to use the
weight of a bushel than it is to use the measure of a
bushel, hence the following weights have been fixed by
law:

1. A bushel of wheat, of peas, of clover seed, of beans,
   or of potatoes *weighs 60 lb.*

2. A bushel of rye, of Indian corn, or of flax seed
   weighs 56 lb.

3. A bushel of barley, of timothy seed, or of buckwheat
   weighs 48 lb.

4. A bushel of beets, of carrots, of onions, or of turnips
   weighs 50 lb.

5. A bushel of oats *weighs 34 lb.*

The following facts are also important:

> 1 barrel of flour weighs 196 lb.
> 1   "   " pork   "   200 lb.

What is the value of:

1. 180 lb. of wheat @ $2·10 a bu.?
2. 150 lb. of wheat @ $2·00 a bu.?
3. 96 lb. of barley @ $1·25 a bu.?
4. 360 lb. of peas @ $3·20 a bu.?
5. 102 lb. of oats @ 85c. a bu.?
6. 90 lb. of clover seed @ $36·00 a bu.?
7. 112 lb. of flax seed @ $8·00 a bu.?
8. 144 lb. of timothy seed @ $12·00 a bu.?
9. 300 lb. of beans @ $8·40 a bu.?
10. 150 lb. of potatoes @ $1·20 a bu.?

EXERCISE 32

What is the value of:

1. A load of wheat weighing 3360 lb. @ $2·05 a bu.?
2. A load of barley weighing 2976 lb. @ $1·35 a bu.?
3. A load of oats weighing 2448 lb. @ $1·02 a bu.?
4. A load of peas weighing 3240 lb. @ $3·50 a bu.?
5. A load of potatoes weighing 3540 lb. @ $2·50 a bu.?
6. A load of beans weighing 3420 lb. @ $8·75 a bu.?
7. A load of flax seed weighing 3024 lb. @ $8·50 a bu.?
8. Three loads of wheat weighing 3090 lb., 3210 lb., and 3420 lb. if wheat is worth $2·12 a bu.?
9. Four loads of oats weighing 2482 lb., 2550 lb., 2669 lb., and 2771 lb., @ 98c. a bu.?
10. What is the value of a bin of barley containing 5760 lb., @ $1·40 a bu.?

TIME MEASURE

Complete the following table, then memorize it:

.... seconds (sec.)	= 1 minute (min.)
.... minutes (min.)	= 1 hour (hr.)
.... hours (hr.)	= 1 day (da.)
.... days (da.)	= 1 week (wk.)
.... months (mo.)	= 1 year (yr.)
.... days	= 1 common year
.... days	= 1 leap-year
.... years	= 1 century.

NOTE 1.—The standard unit for measuring time is the mean solar year, which is equal to 365 days, 5 hr. 48 min. 46 sec., or nearly 365¼ days.

NOTE 2.—Commit to memory the following lines:

Thirty days have September,
April, June, and November;
February has twenty-eight alone—
All the rest have thirty-one;
But leap-year coming once in four,
February then has one day more.

NOTE 3.—Every year whose number is divisible by 4 is a leap-year, unless the number of the year ends in two ciphers (as 1600, 1900), in which case the date must be divisible by 400.

NOTE 4.—Time *before noon* is indicated by a.m., *noon* by m., time *after noon* by p.m.

EXERCISE 33

1. How many hours from 10 a.m. to 3 p.m.?

2. How many hours and minutes from 7 a.m. to 2.30 p.m.?

3. How many hours and minutes from 8 a.m. to 3.15 p.m.?

4. How many hours and minutes from 2.30 p.m. to 8.45 a.m.?

5. How many days from:

      (a) Aug. 9th. to Aug. 25th.?

      (b) July 5th. to Sept. 4th.?

      (c) March 21st. to June 30th.?

      (d) Jan. 23rd. to Oct. 9th. (1919)?

      (e) Jan. 5th. to Dec. 26th. (1920)?

NOTE.—In finding the time between such dates, leave out the first day; for example, from the 3rd. to the 28th. is 25 days.

6. From a calendar find how many working days there are in the month of March? of April? of July? of September? of November?

7. A carpenter is at work from 7 a.m. to 12 m., and from 1 p.m. to 6 p.m. each day for 6 days in the week. If he earns 75c. an hour, how much will he earn in a week?

8. A plumber is engaged on a piece of work from 10 a.m. on Monday till 2 p.m. on Thursday of the next week. His work day is from 8 a.m. to 12 m. and from 1 p.m. to 5 p.m., except on Saturday when it ends at 12 m. If he charges 75c. an hour, how much should he receive?

9.   Time Sheet

Name of Workman	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total Time@	Wages 68c.hr.
John Smith	9	9½	8	10	9½	8		
James Grant	8½	9	7½	9	8¾	7¼		
Robt. Jones	10	9½	8½	7½	10	9½		
Richard Owens	7½	9	10	8	9	10		
Wm. Main	6¼	8½	0	7½	8¾	0		
A. A. James	0	7½	10	8½	0	7		
S. F. Nash	10	10	10	10	10	10		
A. B. Nelson	7	8	9	10	10	10		

(a) Find the week's wages for each workman.

(b) Find the total wages paid to the eight men.

(c) How much did Wm. Main lose by not working full time?

NOTE.—The working day is 10 hours.

10. Make out a weekly time sheet for 5 bricklayers. Wages paid $1·25 an hour, 9-hour day, double pay for overtime.

MISCELLANEOUS TABLE

12 things.	1 dozen (doz.)
12 dozen things.	1 gross (gro.)
12 gross.	1 great gross
20 single things.	1 score
24 sheets of paper. . .	1 quire (qr.)
20 quires.	1 ream (rm.)

EXERCISE 34

1. What is the value of 156 eggs @ 58c. a doz.?

2. Find the cost of 66 lemons @ 40c. a doz.

3. How old is a man who says he has lived three-score and ten years?

4. What will a gross of lead-pencils cost @ 36c. a doz.?

5. What is the cost of a great gross of buttons @ 15c. a doz.?

6. A stationer buys paper @ 40c. a quire and sells it @ 3c. a sheet.   What is his gain on a ream?

7. A dealer gets a dozen boxes of pens, each containing one gross.   What are the pens worth @ 1c. each?

8. At a public school promotion examination there are 40 pupils writing.   If each pupil is examined in 8 subjects and uses 3 sheets for each subject, how many reams of paper are used?

9. A dealer buys 5 gross of copy-books @ 25c. a doz. and sells them @ 3c. each. What is his gain on the lot?

10. A stationer buys 6 doz. writing tablets @ $1·75 a doz. and sells them @ 25c. each. What is his gain on the 6 doz.?

FRACTIONS

ORAL EXERCISE

1. Read the following fractions:

$\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{5}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{5}{8}$, $\frac{3}{7}$, $\frac{7}{9}$, $\frac{9}{10}$, $\frac{6}{11}$, $\frac{4}{10}$, $\frac{3}{5}$, $\frac{3}{10}$, $\frac{7}{15}$, $\frac{9}{20}$, $\frac{85}{100}$, $\frac{47}{100}$, $\frac{75}{100}$, $\frac{27}{100}$.

2. Write the following fractions in figures:

Two thirds; three fifths; four sevenths; five ninths; six tenths; seven eighths; seven fifteenths; nine tenths; eight twentieths; twelve twenty-fifths; forty-two hundredths; sixty-seven hundredths; fourteen fiftieths; eighty hundredths.

3. Take the following numbers for denominators, choose a numerator for each denominator, and read the fractions formed:

5, 8, 7, 6, 9, 12, 15, 14, 17, 20, 25, 100.

4. Take the following numbers for numerators, choose a denominator for each numerator, and read the fractions formed:

3, 6, 4, 7, 8, 9, 10, 11, 14, 17, 15, 18, 20.

5. Into how many equal parts does each of the following fractions show that the whole has been divided?

$\frac{7}{8}$, $\frac{5}{9}$, $\frac{4}{7}$, $\frac{8}{11}$, $\frac{3}{9}$, $\frac{11}{12}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{5}{11}$, $\frac{9}{20}$, $\frac{7}{25}$, $\frac{6}{50}$, $\frac{82}{100}$, $\frac{47}{100}$, $\frac{85}{100}$, $\frac{93}{100}$.

6. How many of the equal parts are taken in each of the fractions in Question (5)?

7. State what each of the following fractions means:

$\frac{3}{5}$, $\frac{4}{9}$, $\frac{7}{13}$, $\frac{8}{15}$, $\frac{9}{16}$, $\frac{6}{13}$, $\frac{7}{20}$, $\frac{8}{35}$, $\frac{11}{50}$, $\frac{65}{100}$, $\frac{88}{100}$.

8. Make a diagram to show $\frac{2}{3}$ of a rectangle, $\frac{4}{5}$ of a rectangle, $\frac{3}{4}$ of a rectangle, $\frac{3}{5}$ of a rectangle, $\frac{9}{10}$ of a rectangle.

9. What part of a foot is 6 inches? is 2 inches? is 8 inches? is 10 inches?

10. What part of a yard is 1 foot? is 2 feet? is 3 feet?

11. What part of $10 is $3? is $5? is $8? is $9?

12. What part of an hour is 10 min.? is 15 min.? is 20 min.? is 30 min.? is 45 min.? is 50 min.?

13. What is $\frac{2}{3}$ of 12? $\frac{3}{4}$ of 20? $\frac{4}{7}$ of 14? $\frac{3}{5}$ of 25? $\frac{4}{9}$ of 27? $\frac{3}{11}$ of 33? $\frac{4}{7}$ of 35? $\frac{5}{12}$ of 48? $\frac{9}{11}$ of 66?

14. What is $\frac{5}{6}$ of $60? $\frac{2}{5}$ of 40 acres? $\frac{5}{12}$ of 48 ft.? $\frac{8}{7}$ of 63 hr.? $\frac{4}{11}$ of 66 dy.? $\frac{3}{4}$ of 36c.? $\frac{2}{3}$ of 36 tons? $\frac{7}{8}$ of 56 mi.? $\frac{3}{8}$ of $72? $\frac{9}{11}$ of $88? $\frac{6}{8}$ of $64?

15. What is the value of:

    (a) $\frac{3}{4}$ lb. of butter @ 44c. a lb.?

    (b) $\frac{3}{8}$ lb. of margarine @ 32c. a lb.?

    (c) 2 pk. of potatoes @ $1·60 a bu.?

    (d) $\frac{2}{3}$ of a ton of coal @ $9 a ton?

    (e) $\frac{5}{6}$ of a bu. of oats @ 84c. a bu.?

    (f) $\frac{6}{7}$ of a yard of cloth @ $1·40 a yard?

    (g) 3 gal. of oats @ 88c. a bu.?

    (h) $\frac{3}{4}$ of an acre of flax @ $140 an acre?

    (i) $\frac{4}{5}$ of a day's work @ $4·50 a day?

    (j) $\frac{2}{5}$ of a barrel of flour @ $10·50 a barrel?

16. A farmer had 200 bu. of wheat. He sold $\frac{2}{5}$ of it at $2 a bu. How much did he receive for the wheat he sold?

17. A farmer had 100 acres on his farm. He had $\frac{1}{2}$ of it in hay, $\frac{1}{4}$ of it in pasture, and $\frac{1}{10}$ of it in hoed crop. How many acres of hay did he have? of pasture? of hoed crop?

ORAL EXERCISE

1. How many halves in $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $5\frac{1}{2}$, $4\frac{1}{2}$, $8\frac{1}{2}$, $6\frac{1}{2}$, $9\frac{1}{2}$, $7\frac{1}{2}$, $10\frac{1}{2}$, $12\frac{1}{2}$? Why?

Write these mixed numbers as improper fractions.

2. How many thirds are there in:

$1\frac{1}{3}$, $2\frac{2}{3}$, $3\frac{1}{3}$, $4\frac{2}{3}$, $5\frac{1}{3}$, $6\frac{2}{3}$, $7\frac{1}{3}$, $7\frac{2}{3}$, $8\frac{2}{3}$, $9\frac{1}{3}$, $9\frac{2}{3}$, $10\frac{1}{3}$, $11\frac{2}{3}$? Why?

Write these mixed numbers as improper fractions.

3. How many fourths are there in:

$2\frac{1}{4}$, $3\frac{3}{4}$, $4\frac{2}{4}$, $5\frac{3}{4}$, $6\frac{1}{4}$, $7\frac{2}{4}$, $8\frac{3}{4}$, $9\frac{2}{4}$, $10\frac{2}{4}$, $11\frac{1}{4}$, $11\frac{2}{4}$, $11\frac{3}{4}$? Why?

4. How many fifths in $2\frac{3}{5}$? How many sixths in $3\frac{5}{6}$? How many sevenths in $7\frac{4}{7}$? How many eighths in $4\frac{7}{8}$? How many ninths in $6\frac{8}{9}$? How many twelfths in $7\frac{5}{12}$? How many fifteenths in $3\frac{3}{15}$?

5. Reduce each of the following mixed numbers to improper fractions:

$3\frac{4}{5}$, $4\frac{2}{5}$, $7\frac{2}{3}$, $6\frac{4}{7}$, $11\frac{3}{10}$, $14\frac{2}{5}$, $16\frac{2}{3}$, $9\frac{1}{11}$, $8\frac{7}{8}$, $9\frac{4}{5}$, $12\frac{5}{8}$, $13\frac{3}{4}$.

6. Write the following improper fractions as mixed numbers:

$\frac{11}{3}$, $\frac{15}{4}$, $\frac{43}{8}$, $\frac{62}{9}$, $\frac{78}{5}$, $\frac{85}{11}$, $\frac{94}{12}$, $\frac{78}{14}$, $\frac{69}{15}$, $\frac{73}{13}$, $\frac{107}{13}$, $\frac{220}{16}$, $\frac{146}{12}$, $\frac{250}{9}$.

7. Jack had three half-dollar pieces. How much money did he have?

8. Mary had five quarter-dollar pieces. How much money had she?

9. A gardener had 15 plots of ground all of the same size, with $\frac{1}{8}$ of an acre in each plot. How many acres of land had he?

10. What is the value of:

(a) $1\frac{7}{8}$ acres of land @ $80 an acre?

(b) $2\frac{3}{4}$ bu. of wheat @ $2·00 a bu.?

(c) $3\frac{2}{3}$ yd. of cloth @ 60c. a yd.?

(d) $4\frac{1}{4}$ pints of milk @ 12c. a qt.?

(e) $2\frac{1}{3}$ lb. of meat @ 30c. a lb.?

(f) $3\frac{1}{2}$ lb. of butter @ 40c. a lb.?

EQUIVALENT FRACTIONS

1. Take a square piece of paper 4 in. to the side. Fold it into two equal parts. What is each equal part called?

2. Fold each half into two equal parts. How many equal parts? What is each part called?

3. Fill in the blanks:

$$\tfrac{1}{2} = \tfrac{}{4}, \tfrac{2}{2} = \tfrac{}{4}.$$

4. Fold a square piece of paper 6 in. to the side into 3 equal parts. What is each equal part called?

5. Fold the paper again so that each third is divided into two equal parts. How many equal parts are there? What is each equal part called?

6. Fill in the blanks:

$$\tfrac{1}{3} = \tfrac{}{6}, \qquad \tfrac{2}{3} = \tfrac{}{6}, \qquad \tfrac{3}{3} = \tfrac{}{6}, \qquad \tfrac{1}{2} = \tfrac{}{6}.$$

7. Fold the paper again so that each sixth is divided into two equal parts. Into how many equal parts is the square now divided? What is each equal part called?

8. Fill in the blanks:

$$\tfrac{1}{3} = \tfrac{}{6} = \tfrac{}{12} \qquad \tfrac{1}{6} = \tfrac{}{12} \qquad \tfrac{1}{2} = \tfrac{}{6} = \tfrac{}{12}$$
$$\tfrac{2}{3} = \tfrac{}{6} = \tfrac{}{12} \qquad \tfrac{3}{6} = \tfrac{}{12} \qquad \tfrac{1}{4} = \tfrac{}{12}.$$
$$\tfrac{3}{3} = \tfrac{}{6} = \tfrac{}{12} \qquad \tfrac{5}{6} = \tfrac{}{12}$$

9. Fill in the following blanks:

$$\tfrac{2}{4} = \tfrac{}{2} = \tfrac{}{8}, \qquad \tfrac{3}{4} = \tfrac{}{8} = \tfrac{}{12} = \tfrac{}{16}$$
$$\tfrac{1}{3} = \tfrac{}{9} = \tfrac{}{12} = \tfrac{}{15} = \tfrac{}{18}$$
$$\tfrac{2}{5} = \tfrac{}{10} = \tfrac{}{15} = \tfrac{}{20} = \tfrac{}{25}$$
$$\tfrac{3}{7} = \tfrac{}{14} = \tfrac{}{21} = \tfrac{}{28} = \tfrac{}{35} = \tfrac{}{42}$$
$$\tfrac{5}{9} = \tfrac{}{18} = \tfrac{}{27} = \tfrac{}{36} = \tfrac{}{45} = \tfrac{}{54} = \tfrac{}{63}$$
$$\tfrac{2}{11} = \tfrac{}{22} = \tfrac{}{33} = \tfrac{}{44} = \tfrac{}{55} = \tfrac{}{66} = \tfrac{}{88}$$
$$\tfrac{5}{12} = \tfrac{}{24} = \tfrac{}{36} = \tfrac{}{48} = \tfrac{}{60} = \tfrac{}{72} = \tfrac{}{84}.$$

10. $\frac{24}{36} = \frac{}{9} = \frac{}{3}$

   $\frac{32}{64} = \frac{}{16} = \frac{}{8} = \frac{}{4} = \frac{}{2}$

   $\frac{48}{72} = \frac{}{36} = \frac{}{18} = \frac{}{9} = \frac{}{3}.$

ORAL EXERCISE

1. How many eighths are there in $\frac{1}{2}$? in $\frac{3}{4}$? in $\frac{8}{16}$? in $\frac{12}{32}$?

2. How many twentieths are there in $\frac{1}{2}$? in $\frac{3}{4}$? in $\frac{2}{5}$? in $\frac{9}{10}$? in $\frac{16}{40}$?

3. Change each of the following fractions to twelfths:

$$\frac{2}{3}, \quad \frac{3}{4}, \quad \frac{2}{8}, \quad \frac{5}{6}, \quad \frac{1}{2}.$$

4. Change each of the following fractions to thirtieths:

$$\frac{3}{5}, \quad \frac{2}{3}, \quad \frac{5}{6}, \quad \frac{7}{10}, \quad \frac{8}{15}.$$

5. Change each of the following fractions to hundredths:

$$\frac{3}{4}, \quad \frac{7}{20}, \quad \frac{9}{10}, \quad \frac{3}{50}, \quad \frac{2}{5}.$$

6. Change each of the following pairs of fractions to equivalent fractions having the same denominator:

$\frac{1}{2}$ and $\frac{3}{4}$, $\frac{1}{4}$ and $\frac{5}{6}$, $\frac{1}{2}$ and $\frac{2}{5}$, $\frac{2}{3}$ and $\frac{3}{4}$,

$\frac{3}{4}$ and $\frac{5}{6}$, $\frac{3}{7}$ and $\frac{3}{5}$.

7. Put in the right numerators in the following:

$\frac{3}{5} = \frac{}{30}$, $\frac{2}{7} = \frac{}{21}$, $\frac{5}{6} = \frac{}{42}$, $\frac{7}{8} = \frac{}{56}$,

$\frac{7}{9} = \frac{}{63}$, $\frac{4}{11} = \frac{}{55}$, $\frac{7}{10} = \frac{}{80}$, $\frac{4}{25} = \frac{}{100}.$

8. Put in the right denominators in the following:

$\frac{5}{7} = \frac{30}{}$, $\frac{4}{6} = \frac{28}{}$, $\frac{3}{8} = \frac{15}{}$, $\frac{9}{13} = \frac{36}{}$,

$\frac{8}{11} = \frac{56}{}$, $\frac{3}{10} = \frac{21}{}$, $\frac{4}{100} = \frac{12}{}$, $\frac{7}{9} = \frac{35}{}.$

ADDITION OF FRACTIONS

EXERCISE 35

1. Find the value of:

(a) $\frac{1}{2} + \frac{2}{2}$ (b) $\frac{2}{3} + \frac{4}{3}$ (c) $\frac{2}{5} + \frac{3}{5}$ (d) $\frac{3}{7} + \frac{2}{7}$

(e) $\frac{4}{9} + \frac{2}{9}$ (f) $\frac{5}{10} + \frac{4}{10}$ (g) $\frac{2}{12} + \frac{3}{12} + \frac{6}{12}$

(h) $\frac{4}{20} + \frac{3}{20} + \frac{7}{20}$ (i) $\frac{3}{50} + \frac{8}{50} + \frac{39}{50}$

2. Find the value of:

(a) $\frac{1}{2} + \frac{1}{4}$ (b) $\frac{1}{2} + \frac{3}{4}$ (c) $\frac{1}{3} + \frac{1}{6}$ (d) $\frac{1}{4} + \frac{3}{8}$

(e) $\frac{1}{5} + \frac{3}{10}$ (f) $\frac{3}{4} + \frac{2}{8}$ (g) $\frac{2}{3} + \frac{4}{9}$ (h) $\frac{3}{5} + \frac{7}{10}$

           (i) $\frac{1}{5} + \frac{3}{10} + \frac{7}{20}$

Example:

Find the value of: $\frac{2}{3} + \frac{3}{4}$

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18}$ etc.

$\frac{3}{4} = \frac{6}{8} = \quad\ \ \frac{9}{12} = \frac{12}{16} = \frac{15}{20}$ "

$\therefore \ \ \frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}.$

3. Find the value of:

(a) $\frac{3}{4} + \frac{5}{6}$ (b) $\frac{4}{5} + \frac{2}{3}$ (c) $\frac{3}{5} + \frac{2}{4}$

(d) $\frac{5}{6} + \frac{4}{5}$ (e) $\frac{2}{7} + \frac{1}{2}$ (f) $\frac{3}{8} + \frac{4}{7}$

(g) $\frac{5}{8} + \frac{2}{3}$ (h) $\frac{6}{7} + \frac{2}{9}$ (i) $\frac{4}{5} + \frac{6}{11}$

4. Find the value of:

(a) $\frac{1}{2} + \frac{1}{5} + \frac{1}{3}$ (b) $\frac{1}{4} + \frac{1}{3} + \frac{1}{6}$ (c) $\frac{2}{3} + \frac{1}{4} + \frac{2}{5}$

(d) $\frac{3}{4} + \frac{2}{3} + \frac{1}{6}$ (e) $\frac{4}{5} + \frac{1}{2} + \frac{3}{10}$ (f) $\frac{3}{4} + \frac{2}{3} + \frac{1}{8}$

(g) $\frac{2}{3} + \frac{3}{4} + \frac{5}{8}$ (h) $\frac{1}{2} + \frac{3}{5} + \frac{2}{7}$ (i) $\frac{1}{4} + \frac{2}{3} + \frac{5}{7}$

(j) $\frac{4}{9} + \frac{2}{3} + \frac{1}{2}$ (k) $\frac{5}{7} + \frac{1}{2} + \frac{9}{14}$ (l) $\frac{2}{5} + \frac{1}{4} + \frac{4}{3}$

5. Find the sum of:

$$\$\tfrac{3}{4}, \ \$\tfrac{7}{8}, \ \$\tfrac{2}{3}, \ \text{and} \ \$\tfrac{3}{8}.$$

6. Jack had $\frac{1}{4}$ of a dollar. Alex had $\frac{6}{8}$ of a dollar. How much money did they together have?

7. A grocer sold $\frac{5}{8}$ of a lb. of sugar to one customer and $\frac{3}{4}$ of a lb. to another. How many lb. did he sell?

8. A boy had $\$\frac{4}{5}$ left after paying $\$\frac{1}{4}$ for a book. How much money had he at first?

9. A gardener had $\frac{1}{4}$ acre of tomatoes, $\frac{1}{6}$ acre of beans, and $\frac{1}{3}$ acre of peas. How much land did he have altogether?

10. A milkman sold $\frac{1}{4}$ gal. of milk to one customer and $\frac{2}{3}$ gal. to another. How much milk did he sell?

11. A boy spent $\$\frac{3}{4}$ for a hat, $\$\frac{9}{10}$ for a ball, and $\$\frac{4}{5}$ for a glove. How much did he spend?

EXERCISE 36

1. To add $\$5\frac{1}{2}$ and $\$3\frac{1}{4}$:

$\$5 + \$3 = \$8$ $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\$8 + \$\frac{3}{4} = \$8\frac{3}{4}.$

2. Find the value of:

$$(a) \quad 4\frac{3}{4} + 2\frac{7}{8}$$
$$(b) \quad 5\frac{3}{7} + 4\frac{8}{9}$$
$$(c) \quad 4\frac{5}{6} + 3\frac{7}{9}$$
$$(d) \quad 2\frac{1}{2} + 3\frac{3}{4} + 4\frac{1}{4}$$
$$(e) \quad 3\frac{2}{3} + 2\frac{1}{7} + 4\frac{1}{2}$$
$$(f) \quad 2\frac{3}{5} + 1\frac{4}{5} + 5\frac{3}{8}.$$

3. A farmer sold 2 stacks of hay, in one of them there were $8\frac{1}{4}$ tons and in the other $6\frac{1}{3}$ tons. How many tons of hay in the two stacks?

4. There are $82\frac{3}{4}$ acres left in a farm after $9\frac{1}{2}$ acres were sold. How many acres were in the farm at first?

5. A playground is $16\frac{1}{3}$ yd. wide and $25\frac{2}{3}$ yd. long. Find the distance around it.

6. A dealer bought oats at $83\frac{1}{2}$c. a bushel, and in selling it made a gain of $9\frac{3}{4}$c. a bushel. What was the selling price a bushel?

7. Harry worked $11\frac{1}{4}$ hours on Saturday and Jim $12\frac{1}{3}$. How many hours did they both work?

8. A lady bought three pieces of ribbon. The first contained $2\frac{1}{3}$ yd., the second $3\frac{1}{4}$ yd., and the third $4\frac{5}{8}$ yd. How many yards of ribbon did she buy?

6

SUBTRACTION OF FRACTIONS

EXERCISE 37

1. Find the value of:

(a) $\frac{4}{5} - \frac{2}{5}$ (k) $\frac{3}{4} - \frac{1}{3}$

(b) $\frac{8}{9} - \frac{3}{9}$ (l) $\frac{4}{5} - \frac{1}{2}$

(c) $\frac{8}{12} - \frac{5}{12}$ (m) $\frac{6}{7} - \frac{3}{4}$

(d) $\frac{19}{20} - \frac{13}{20}$ (n) $\frac{8}{9} - \frac{3}{7}$

(e) $\frac{90}{100} - \frac{65}{100}$ (o) $\frac{7}{12} - \frac{2}{8}$

(f) $\frac{3}{4} - \frac{1}{2}$ (p) $\frac{11}{15} - \frac{3}{8}$

(g) $\frac{7}{8} - \frac{3}{4}$ (q) $\frac{7}{8} - \frac{2}{3}$

(h) $\frac{11}{12} - \frac{3}{8}$ (r) $\frac{4}{5} - \frac{2}{7}$

(i) $\frac{8}{15} - \frac{2}{5}$ (s) $\frac{11}{12} - \frac{8}{11}$

(j) $\frac{19}{20} - \frac{3}{5}$ (t) $\frac{7}{9} - \frac{3}{4}$.

2. What must be added to $\frac{3}{7}$ to make $1\frac{1}{2}$?

3. What is the difference between $$\frac{3}{8}$ and $$\frac{5}{8}$?

4. In one lot there is $\frac{3}{5}$ of an acre, in another $\frac{3}{8}$ of an acre. How much larger is one lot than the other?

5. Find the difference between $(\frac{2}{3} + \frac{1}{4})$ and $(\frac{3}{4} + \frac{1}{6})$.

EXERCISE 38 (MISCELLANEOUS)

1. Add and check:

(a) $47259·63	(b) $56437·82	(c) $476385·79
28754·37	9642·75	28946·43
97568·46	908·36	70·08
85947·38	89375·42	875·98
29568·48	8·07	9376·43
48936·74	48639·59	82965·78
98765·33	63925·03	146893·62
49876·54	78·67	784327·59
38295·73	859·43	5936·46

2. Subtract and check:

(a) $408960·36	(b) $820400·38	(c) $786500·32
289794·29	265948·71	297643·87

3. Find the value of and check:

    (*a*) 86593 × 468   (*b*) 92764 × 589
          (*c*) 468325 × 673

4. Find the value of and check:

    (*a*) 632897 ÷ 364   (*b*) 9383654 ÷ 2946
          (*c*) 476389 ÷ 5863

5. Find the value of, using factors:

    (*a*)   594036 ÷ 18   (*b*)   768375 ÷ 45
    (*c*) 4763808 ÷ 24   (*d*) 1989204 ÷ 42

6. Find the value of:

    (*a*) 15 × 18 × 23   (*b*) 16 × 14 × 36
    (*c*) 14 × 23 × 56   (*d*) 29 × 23 × 39

7. Make out a bill for the following sales. Supply name of purchaser, seller, date, place, etc. Receipt the bill:

        1½ lb. ham @ 42c. a lb.
        9 cans of tomatoes @ $1·20 a doz.
        2¼ lb. tea @ 56c. a lb.
        3¾ lb. cheese @ 24c. a lb.

8. Make out a bill containing 5 items such as one would receive from a hardware merchant.

9. Mary made a strawberry shortcake for which she used 3 cups of flour (1½ pt.) @ 6c. a pint, 2 teaspoonfuls of baking-powder worth ⅓c. each, ¼ lb. of butter @ 44c. a lb., ½ pt. of milk @ 6c. a pt., ½ pt. of cream @ 60c. a qt., and 1 box of strawberries @ 23c. a box. Find the cost of the cake.

10. A coal dealer delivered 5 loads of coal weighing: 2867 lb., 3232 lb., 2964 lb., 3105 lb., and 3002 lb. Find the average weight of the loads.

11. A grocer's sales for 6 days of a week were: $74·38, $65·92, $87·45, $75·59, $69·32, and $83·64. Find the average daily sales for the week.

12. Find the difference between the value of a load of wheat containing 52 bu. @ $2·20 a bu. and a load of oats containing 68 bu. @ 82c. a bushel.

13. How many pence in £3 5s. 6d.?

14. Find the value of:

      (a) A load of barley containing 2976 lb. @ 95c. a bu.

      (b) A load of oats containing 2499 lb. @ 80c. a bu.

      (c) A load of potatoes containing 2655 lb. @ $1·20 a bu.

      (d) A load of wheat containing 2910 lb. @ $2·00 a bu.

      (e) A load of flax seed containing 2800 lb. @ $6·00 a bu.

      (f) A load of hay containing 3500 lb. @ $12 a ton

      (g) A load of peas containing 2670 lb. @ $3 a bu.

15. Quart bottles are filled from a 54-gal. cask of wine. How many bottles will be filled and what are they worth at $2.50 a bottle?

16. A horse is given 3 feeds of oats a day, each consisting of one gal. How long will 27 bu. feed him?

17. A farmer's wife sold 15¾ lb. of butter @ 32c. a lb. and got in exchange for it molasses @ 63c. a gal. How many gal. did she get?

18. Find the cost of 4 pk. 5 qt. 1 pt. of berries @ 14c. a quart.

19. In 2940 lb. of flour, how many barrels?

20. A grocer bought potatoes @ $1·20 a bu. and sold them @ 50c. a pk. How much did he gain on 425 bu.?

21. Find the cost of the beans required to plant 5 acres, allowing 12 quarts to the acre, the beans being worth $1·50 a pk.

22. If a woodpecker eats on an average 100 insect pests in a day, how many will 250 woodpeckers eat during the month of May?

23. What is $\frac{7}{8}$ of a bu. of wheat worth @ $2·08 a bu.?

24. What is $1\frac{1}{2}$ of an acre of land worth @ $84 an acre?

25. How many lb. in $1\frac{3}{7}$ of a bu. of oats?

26. Find the amount of the following threshing bill:

> 760 bu. of wheat @ 5c. a bu.
> 2250 bu. of oats @ 3c. a bu.
> 525 bu. of barley @ 4c. a bu.
> 90 bu. of peas @ 5c. a bu.
> 2 tons of coal @ $11·50 a ton
> 8 men for $1\frac{1}{2}$ days at $3·00 per day.

27. A farmer had 220 bu. of wheat. He sowed 75 bu., sold $26\frac{1}{4}$ bu., and used $15\frac{3}{4}$ bu. How many bu. has he left?

28. Make out in proper form the following account:

> James Smith of Winnipeg buys of Simon Grey, merchant of the same city, the following goods:

> May 3rd, 1919:

> 12 yards dress goods @ $1·75 a yd.

> May 10th. 25 yards linen @ 85c. a yd.
> "   "   15   "   gingham @ 27c. a yd.
> " 15th. 18   "   flannel @ 42c. a yd.

> On May 28th. James Smith pays $25 on account.

> On May 31st. James Smith pays the balance of the account.

29. Reduce $3\frac{1}{2}$, $5\frac{1}{4}$, $6\frac{2}{3}$, $7\frac{5}{8}$, and $8\frac{2}{5}$ to improper fractions.

30. Supply the correct numerators for the following fractions:

$\frac{2}{3} = \frac{}{15}$, $\frac{4}{7} = \frac{}{21}$, $\frac{5}{8} = \frac{}{24}$, $\frac{7}{8} = \frac{}{48}$.

31. Add $7\frac{2}{3}$, $5\frac{4}{5}$, and $2\frac{1}{4}$.

32. Find the value of (a) $\frac{4}{5} - \frac{2}{7}$, (b) $\frac{8}{9} - \frac{7}{11}$, (c) $\frac{11}{12} - \frac{4}{5}$.

SECTION III

ORAL EXERCISE

To add 32 and 43 proceed as follows:

$$32 + 3 = 35; 35 + 40 = 75.$$

Find the value of:

1. $32 + 43$	11. $68 + 52$
2. $43 + 25$	12. $49 + 75$
3. $26 + 33$	13. $63 + 68$
4. $54 + 25$	14. $68 + 53$
5. $63 + 24$	15. $56 + 78$
6. $47 + 24$	16. $85 + 47 + 20$
7. $64 + 37$	17. $93 + 89 + 30$
8. $59 + 27$	18. $39 + 69 + 45$
9. $38 + 48$	19. $47 + 63 + 68$
10. $56 + 45$	20. $85 + 75 + 33$

ORAL EXERCISE

Use the foregoing method to add two columns of the following addition questions at the same time:

(1) 42	(2) 53	(3) 46	(4) 74	(5) 85	(6) 65
76	45	53	28	73	47
34	63	75	59	28	38
28	74	28	47	54	92
—	—	—	—	—	—

(7) 74	(8) 64	(9) 75	(10) 94	(11) 39	(12) 59
26	57	46	53	54	84
59	38	83	76	65·	63
83	93	72	84	74	72
—	—	—	—	—	—

(13) 56	(14) 49	(15) 54	(16) 74	(17) 65	(18) 78
47	38	63	53	74	63
38	67	75	89	39	82
26	49	94	47	47	75
55	58	86	38	65	40
—	—	—	—	—	—

EXERCISE 39

1. Subtract and test the following six problems in three minutes:

(a)	542836	(b)	493624	(c)	928473
	289479		289789		289768

(d)	8427635	(e)	2840050	(f)	8400206
	2989948		1968249		2759478

2. Find the value of and test one problem in two minutes:

    (a) 4897×6854 (d) 9876×6543

    (b) 7593×8574 (e) 9278×5937

    (c) 8794×6547 (f) 8792×3456

3. Find the value of and test:

    (a) 6894739 by 74638   (b) 9276839 by 82965

    (c) 7864938 by 95876   (d) 8279643 by 94673

ACCOUNTS

THE CASH ACCOUNT

Every person, whether in business or not, who receives and pays out money, should keep a **cash account.** Even school-boys and school-girls who are earning or receiving money should keep a strict account of their receipts and expenditures. It is a good lesson in business to keep such an account.

The following is a convenient form for a cash account:

Harry Williams' Cash Account

		Receipts						Payments		
1920					1920					
Jan.	1	Cash on hand	7	35	Jan.	5	For a hockey stick			75
"	5	Delivering papers	2	80	"	9	For a pr. of boots		4	25
"	13	Running errands		65	"	20	Ticket for theatre			11
"	24	Shovelling snow		35	"	30	Necktie			75
"	31	Delivering bills		30	"	31	Ticket for hockey			
							game			25
							Balance on hand		5	34
			11	45					11	45
1920										
Feb.	2	Balance	5	34						

NOTES.—1. That a double line in the middle of the account divides it. On the *left* hand side of this line are entered the *cash receipts*, and on the *right* hand side the *cash payments*.

2. For each item we require the date, the amount, and for what the cash was received or paid out.

3. A cash account should be balanced at least monthly. The balance should be written in red ink, and should be carried forward to the next month.

EXERCISE 40

1. Make out cash accounts for each of the following months:

February—

(a) Feb. 1. Cash on hand—$7·67

Feb. 3, Ran errands, 55c.; Feb. 4th, Paid for having skates sharpened, 15c.; Feb. 7th, Received for work after school, 25c.; Feb. 10th, Bought pair of skate straps, 20c.; Feb. 10th, Received for delivering papers, $3·00; Feb. 18th, Received for sawing wood, 35c; Feb. 24th, Bought a pair of gloves, $1·45; Feb. 28th, Shovelling snow, 15c.

(b) *March*—

March 1. Cash on hand—   What is the balance
for February?

March 4th, For shovelling snow, 40c; For running
errands, 25c.; March 6th, For delivering papers,
$2·80; March 10th, For sawing kindling wood, 50c.;
March 12th, Bought a cap, $1·00; March 20th,
Bought a hockey stick, 60c.; March 29th, Bought
a knife, 45c.; March 31st, Bought a pair of socks, 35c.

(c) *April*—

April 1. Cash on hand—

April 2nd, Selling papers, 30c.; Delivering papers,
$2·80; April 5th, For work, 90c.; April 9th, Bought
a book, 65c.; April 15th, For splitting wood, 75c.;
April 20th, Bought a fish-line and hooks, 35c.; April
30th, Bought fireworks, 50c.; For running errands,
25c.

(d) *May*—

May 1. Cash on hand—

May 3rd, Delivering papers, $3·25; Running errands,
15c.; May 6th, Bought fireworks, 75c.; May 10th,
For work done, 80c.; May 18th, For cleaning yard,
50c.; May 22nd, For digging garden, 75c.; May 26th,
For planting potatoes, 60c.; May 28th, For mowing
lawn, 25c.; May 30th, Bought a pair of baseball
boots, $1·35; May 31st, Bought a baseball, $1·25.

(e) *June*—

June 1. Cash on hand—

June 2nd, Delivering papers, $3·50; June 3rd,
Delivering handbills, 25c.; Mowing lawn, 15c.;
June 5th, Weeding in garden, 35c.; June 10th,
Hilling potatoes, 20c.; June 20th, Bought a suit, $9.50;
June 25th, Bought a straw hat, $1·10; June 28th,
Bought 6 handkerchiefs @ 15c.; June 30th, Spray-
ing garden, 50c.; mowing lawn, 25c.; Bought a
pair of boots, $3·75.

2. Supply your own items for a cash account for the
month of July.

3. Make out the following farmer's cash account for the months of July and August. Balance the account on July 31st and Aug. 31st.

(a) July 1st. Cash on hand—$324·75
        3rd. Paid cash for a binder, $215.00
        5th. Received cash for eggs, $19·65
        7th. Received for load of wheat, cash $135·75
        10th. Paid month's wages to hired man, $45·00
        14th. Paid cash for groceries, $26·35
        18th. Received cash for eggs, $25·19
        22nd. Received cash for milk sold, $139·60
        28th. Paid for extra help, $9·00
        29th. Sold for cash a load of hogs, $362·75
        30th. Paid cash for 2 cows, $225·00.

(b) Aug. 1st. Cash on hand—
        3rd. Paid for one day's threshing, $22·00
        7th. Paid for extra help, $11·50
        9th. Received cash for eggs, $18·75
        10th. Paid month's wages to hired man, $45·00
        15th. Received cash for milk sold, $128·95
        19th. Paid for provisions, $39·75
        20th. Received cash for eggs, $22·32
        21st. Sold beef cattle, $524·50
        25th. Paid cash for a horse, $137·00
        30th. Paid cash for repairs to barn, $139·47
        31st. Paid for extra help, $15·75

PERSONAL ACCOUNTS

A merchant who does not do a strictly cash business must keep a set of books. One of these books is called the Ledger. In it he will have an account for every one to whom he has sold goods *on credit*. For instance, if the merchant sells goods to John Smith, who does not pay cash for them, the merchant will mark *charge* on the

bills of goods received by Mr. Smith and will afterwards enter each item of the different bills in Mr. Smith's account. A Ledger account headed by a person's name is called a **personal account.** One form of a personal account has already been shown in Section II. The following account of John Smith's will illustrate another form which is very often used:

Dr.	John Smith						Cr.		
1920				1920					
Mar. 8	Nails	8	75	Mar. 30	By Cash			50	00
11	Doors	18	50	" 31	By Cash			50	00
19	Door trimmings	12	48	" 31	By Balance			15	53
25	Windows	75	80						
		115	53					115	53

Apr. 1   Balance   15·53

The above is John Smith's account at a hardware store for the month of March, 1920. On the *debtor* side (Dr.) of the account are placed the items of goods purchased by Mr. Smith, and on the *creditor* side (Cr.) are placed the amounts paid by Mr. Smith for the goods which he bought.

These personal accounts are balanced at regular intervals, by footing the debit side and the credit side and subtracting the smaller from the greater. The difference, called the **balance,** is then entered on the side having the *smaller* amount and carried forward on the *opposite* side, to the account of the following month. Why?

The person who sells the goods is the *creditor;* the person who purchases the goods is the *debtor.*

In common usage, the term debtor means any one who owes a debt, and the term creditor means any one to whom a debt is owed.

When you give a person credit, you are said to sell *on account*.

From the following transactions make out N. Moyer's personal account. Balance the account on June 30th.

Stratford—June 1st. Sold N. Moyer 75 bu. oats @ $1·05

10th. Sold N. Moyer 5 bu. wheat @ $3·20

15th. Received from N. Moyer cash $90

16th. Sold N. Moyer 1 ton bran, $54·00

17th. N. Moyer did draying for me, $25·00

20th. Sold N. Moyer 10 bu. barley @ 95c.

28th. Received from N. Moyer cash, $15·00.

2. Make out G. Mills' account. Balance it on Oct. 31st.

Hamilton—Oct. 1st. Bought from G. Mills 10 bags potatoes at $2·25

12th. Bought from G. Mills 25 chickens @ $1·00

15th. Sold G. Mills 2 calves @ $35·00

16th. Bought from G. Mills 1 bbl. apples @ $3·50

21st. Bought from G. Mills 10 bu. wheat @ $3·25

28th. Sold G. Mills a cream separator. $67·50

31st. G. Mills paid cash, $25·00.

3. Make out N. Smith's account. Balance it on July 31st.

Peterboro—July 1st.   Sold N. Smith 1 binder, $230

15th.   N. Smith paid me cash $100

16th.   Sold N. Smith 1 hayloader, $125

18th.   Received from N. Smith 100 bu. oats @ $1·08 and 1 horse $130

27th.   Sold N. Smith a hayrake, $47.00

30th.   N. Smith paid in cash, $150.

4. Make out A. McCabe's account.   Balance it on Nov. 30th.

Ottawa, Ont.—Nov. 1st.   Sold A. McCabe 1 set tires, $125

7th.   Sold A. McCabe 6 spark plugs @ 90c.

10th.   Sold A. McCabe 8 gal. gasoline @ 36c.

13th.   Bought a cow from A. McCabe, $118

15th.   Sold A. McCabe repairs for his car, $27·50

18th.   Vulcanized a tire for A. McCabe, $7·30

27th.   A. McCabe paid in cash, $40

30th.   Sold A. McCabe a spring for car, $18·00.

5. Make out F. Hill's account:

London—Dec. 1st.   Sold F. Hill groceries, $17·55

2nd.   Bought of F. Hill 12 bags of potatoes @ $2·75

6th.   Sold F. Hill meat, $10·85

10th.   Sold F. Hill 2 bu. timothy seed @ $8·75

11th.   Bought from F. Hill 25 lb. butter @ 65c.

15 doz. eggs @ 70c.

12 chickens @ $1·25

28th.   Sold F. Hill groceries, $18·95.

Balance the account on Dec. 31st.

FACTORS, COMMON FACTOR, HIGHEST COMMON FACTOR

What is the common factor of 21 and 35?
What are the common factors of 16 and 24?
Which of these common factors is the greatest?

The greatest factor common to two or more numbers is called their **highest common factor** (H.C.F.) or **greatest common measure** (G.C.M.)

When two numbers have no common factor greater than 1, they are said to be **prime to each other.**

What is a prime number?   A prime factor?
Name the prime numbers less than 15.
Name the prime factors of 36, 42, 54, 72.

By using the prime factors of numbers we can readily obtain the H.C.F. of these numbers.

To find the H.C.F. of 36 and 48.

$$36 = 2 \times 18 \qquad\qquad 48 = 2 \times 24$$
$$= 2 \times 2 \times 9 \qquad\qquad = 2 \times 2 \times 12$$
$$= 2 \times 2 \times 3 \times 3. \qquad = 2 \times 2 \times 2 \times 6$$
$$= 2 \times 2 \times 2 \times 2 \times 3.$$

It is easily seen that the highest common factor of these two numbers is $2 \times 2 \times 3 = 12$.

EXERCISE 42

By using prime factors, find the H.C.F. of:

1. 45 and 75
2. 72 and 108
3. 84 and 132
4. 153 and 374
5. 272 and 425
6. 13, 52 and 91
7. 60, 84 and 96
8. 455 and 728
9. 928 and 1073
10. 1536 and 3584
11. 2108 and 3813
12. 112, 128 and 192.

DIVISION BY FACTORS

In Section II of this book it was shown how to divide by the factors of a divisor instead of by the divisor itself, when no remainders occur. It remains to show how to divide by the factors of a divisor when remainders occur.

Example: To divide 73205 by 72, using factors:

$$
\begin{array}{r}
9)\overline{73205} \\
8)\overline{8133}—8 \\
\overline{1016}—5
\end{array}
$$

The division by 9 gives 8133 groups of 9 and a remainder of 8 *ones*.

The division of 8133 groups of 9 by 8 gives 1016 groups of 72 and a remainder of 5 groups of 9.

∴ The complete remainder is 5 groups of 9 plus 8 ones

$$= 45 + 8 = 53.$$

The quotient is 1016 and the remainder is 53.

EXERCISE 43

Using factors, divide:

1. 12409 by 21; by 22; by 24; by 25.
2. 70054 by 36; by 40; by 42; by 44; by 48.
3. 46279 by 27; by 33; by 56; by 72; by 84.
4. 70654 by 105, using 3 factors.
5. 359240 by 385, using 3 factors.

CANCELLATION

Divide 84 by 28, the quotient is 3.

Now divide 84 and 28 each by a common factor 7. The quotients are 12 and 4.

Divide 12 and 4 each by the common factor 4. The quotients are 3 and 1.

Dividing 3 by 1, the quotient is 3, the same as the first quotient.

This work might be put down as follows:

$$84 \div 28 = \frac{84}{28} = \frac{84 \div 7}{28 \div 7} = \frac{12}{4} = \frac{12 \div 4}{4 \div 4} = \frac{3}{1} = 3.$$

Or in shorter form as:

$$\frac{\overset{\overset{\overset{3}{\cancel{12}}}{\cancel{84}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{28}}}} = \frac{3}{1} = 3.$$

This method of obtaining a quotient is called **cancellation.** This method is used in the solution of many problems, because it is easy, quick, and accurate.

EXERCISE 44

1. By cancellation find the quotients of the following:

(a) $\dfrac{105}{35}$　　(h) $\dfrac{560}{420}$　　(o) $\dfrac{81 \times 56}{63 \times 36}$

(b) $\dfrac{132}{66}$　　(i) $\dfrac{1080}{810}$　　(p) $\dfrac{7 \times 9 \times 12 \times 15}{8 \times 15 \times 18 \times 14}$

(c) $\dfrac{256}{64}$　　(j) $\dfrac{204}{85}$　　(q) $\dfrac{12 \times 14 \times 25}{24 \times 35 \times 28}$

(d) $\dfrac{189}{21}$　　(k) $\dfrac{63 \times 78}{42 \times 36}$　　(r) $\dfrac{33 \times 48 \times 56}{22 \times 30 \times 44}$

(e) $\dfrac{225}{45}$　　(l) $\dfrac{84 \times 60}{35 \times 48}$　　(s) $\dfrac{95 \times 105 \times 150}{19 \times 35 \times 50}$

(f) $\dfrac{192}{72}$　　(m) $\dfrac{96 \times 99}{36 \times 88}$　　(t) $\dfrac{8 \times 48 \times 56 \times 81 \times 52}{78 \times 112 \times 27 \times 32}$

(g) $\dfrac{320}{280}$　　(n) $\dfrac{75 \times 49}{35 \times 21}$

2. How many bushels of wheat at $2·40 a bushel will pay for 2 automobiles, each worth $1200?

3. How many bushels of potatoes @ $1·50 per bushel should be given in exchange for 35 bushels of barley @ 90c. a bushel?

4. How many bushels of oats @ 84c. a bushel should be given in exchange for 72 bushels of wheat @ $2·10 a bushel?

5. There were 9 farms, each containing 72 acres and worth $75 an acre, which were exchanged for houses worth $1800 each. How many houses were required?

6. How many tubs of butter, each containing 56 lb. and worth 55c. a lb., are equal in value to 7 tons of hay @ $22 a ton?

MULTIPLES

Name a number which contains 3 an integral number of times; a number which contains 5 an integral number of times; a number which contains 6 an integral number of times.

When one number contains another an integral, or an exact, number of times, the former is said to be a **multiple** of the latter.

Thus 15 is a multiple of 5, because $15 \div 5 = 3$, and 3 is an integer; 24 is a multiple of 6, because $24 \div 6 = 4$, and 4 is an integer.

Name some multiples of 4, of 5, of 7, of 9, of 12.

Name a multiple of 3 which is also a multiple of 4, a multiple of 5 which is also a multiple of 7, a multiple of 6 which is also a multiple of 8.

A number which is a multiple of two or more numbers is called a **common multiple** of those numbers.

Write out the first twelve multiples of 3.

7

Write out the first nine multiples of 4.

Select the common multiples of 3 and 4.

Which of these common multiples is the least?

The least common multiple (L.C.M.) of two or more numbers is the smallest number which will contain each of the numbers an integral number of times.

ORAL EXERCISE

1. Name the first 6 multiples of 5, of 7, of 3.

2. Name the first 8 multiples of 4 and of 6.

3. Name the first 3 common multiples of 4 and 6. What, therefore, is the L.C.M. of 4 and 6?

4. Name the L.C.M. of 5 and 3. Name their next two common multiples.

5. When the L.C.M. of two or more numbers is found, how can other common multiples be found?

6. What must be the prime factors of the L.C.M. of 2, 3, and 5? Why? What must be the prime factors of the L.C.M. of 4, 7, and 9? Why? Then how can the L.C.M. of numbers which are *prime to each other* be found?

7. What is the L.C.M. of 2, 4, 6, and 8? Why is this the same as the L.C.M. of 6 and 8? What is the L.C.M. of 3, 5, and 10? Why is this the same as the L.C.M. of 3 and 10? Then in finding the L.C.M. of 3, 4, 8, and 9, of which of these numbers is it necessary to take account? Why?

The *prime factor method*, therefore, can be used to find the L.C.M. of two or more numbers.

Example: To find the L.C.M. of 24, 30, and 36:

$$24 = 2 \times 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3.$$

It is evident that the L.C.M. of these three numbers must contain all of their *different* prime factors 2, 3, and 5, and that each of these factors must be repeated as many times and no more, as it is repeated in that number in which it occurs the greatest number of times. Thus 2 must be used 3 times, 3 twice, and 5 once. Hence the required L.C.M. is $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

The work may be set down thus:

```
2)24, 30, 36
  2)12 15 18
    3)6 15  9
      2  5  3     L.C.M.=2×2×3×2×5×3=360.
```

EXERCISE 45

Find the L.C.M. of the following:

1. 16, 18, 20	7. 12, 15, 16, 18
2. 12, 28 and 64	8. 22, 77, 143
3. 27, 36 and 42	9. 8, 12, 16, 24, 36
4. 22, 25, 30 and 33	10. 26, 39, 52 and 65
5. 15, 30, 42 and 75	11. 36, 60, 65 and 78
6. 12, 14, 16, 32 and 50	12. 240, 480 and 960.

FRACTIONS

DEFINITIONS

A **fraction** is one or more of the equal parts of a unit, thus $\$\frac{3}{4}$ means that the unit $1 has been divided into 4 equal parts (each $\$\frac{1}{4}$), and that three of these are taken.

What is the meaning of $\$\frac{7}{8}$? of $\frac{9}{11}$ of an apple? of $\frac{14}{15}$ of an acre? of $\frac{24}{25}$ of a bushel?

A fraction may also be considered a denominate number, the unit for which is one of the equal parts of a

larger unit. Just as we call three quarts a denominate number for which the unit is one quart, so three fourths is a denominate number for which the unit is one fourth.

A fraction may also be considered as a quotient, for example, $\frac{4}{5}$ means $4 \div 5$.

Fractions may be written in several ways. For instance, instead of writing "seven eighths" we may write 7 eighths, or $\frac{7}{8}$, all of which are read in the same way. The form "$\frac{7}{8}$" is the one generally used.

In a common fraction there are two numbers, one *above* and the other *below* a *horizontal* line.

These two numbers are called the **terms** of the fraction.

The term below the line is called the **denominator** and the term above the line is called the **numerator.**

Name the terms—the numerator and the denominator —of each of the following fractions:

$$\frac{4}{7}, \quad \frac{3}{8}, \quad \frac{7}{9}, \quad \frac{11}{12}, \quad \frac{15}{20}, \quad \frac{42}{100}, \quad \frac{75}{100}.$$

IMPROPER FRACTIONS AND MIXED NUMBERS

A fraction whose numerator is less than its denominator is a **proper** fraction, for example,

$$\frac{2}{9}, \quad \frac{1}{16}, \quad \frac{16}{17}.$$

The value of a proper fraction is always less than *one*.

A fraction whose numerator equals or exceeds its denominator is an **improper** fraction, for example,

$$\frac{5}{5}, \quad \frac{9}{8}, \quad \frac{24}{12}.$$

The value of an improper fraction is either equal to or greater than *one*.

A number which is composed of an integer and a fraction is a **mixed** number, for example,

$$3\tfrac{1}{2}, \quad 5\tfrac{3}{4}, \quad 6\tfrac{2}{3}.$$

EXERCISE 46

Reduce to improper fractions:

1. $17\tfrac{3}{8}$	7. $26\tfrac{8}{15}$	13. $129\tfrac{4}{11}$
2. $15\tfrac{3}{4}$	8. $45\tfrac{10}{11}$	14. $55\tfrac{35}{44}$
3. $29\tfrac{13}{14}$	9. $57\tfrac{3}{14}$	15. $248\tfrac{5}{8}$
4. $25\tfrac{5}{13}$	10. $61\tfrac{7}{8}$	16. $383\tfrac{7}{15}$
5. $59\tfrac{7}{8}$	11. $125\tfrac{1}{30}$	17. $74\tfrac{4}{5}$
6. $171\tfrac{3}{5}$	12. $167\tfrac{5}{8}$	18. $138\tfrac{10}{11}$

EXERCISE 47

Reduce to mixed numbers:

1. $\tfrac{459}{25}$	5. $\tfrac{1612}{20}$	9. $\tfrac{902}{88}$
2. $\tfrac{393}{14}$	6. $\tfrac{989}{45}$	10. $\tfrac{5973}{66}$
3. $\tfrac{717}{27}$	7. $\tfrac{879}{37}$	11. $\tfrac{7934}{79}$
4. $\tfrac{785}{41}$	8. $\tfrac{838}{18}$	12. $\tfrac{8475}{29}$

EQUIVALENT FRACTIONS
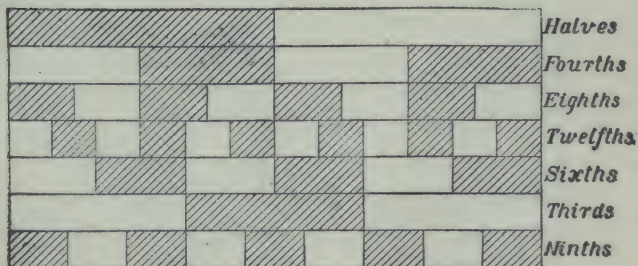
1. With the help of the above diagram, fill in the blanks in (a) to (g).

(a) $\frac{1}{2} = \frac{}{4} = \frac{}{6} = \frac{}{8} = \frac{}{12}$

(b) $\frac{1}{4} = \frac{}{8} = \frac{}{12}$

(c) $\frac{3}{4} = \frac{}{8} = \frac{}{12}$

(d) $\frac{1}{3} = \frac{}{6} = \frac{}{9} = \frac{}{12}$

(e) $\frac{2}{3} = \frac{}{6} = \frac{}{9} = \frac{}{12}$

(f) $\frac{5}{6} = \frac{}{12}$

(g) $\frac{2}{2} = \frac{}{4} = \frac{}{6} = \frac{}{8} = \frac{}{9} = \frac{}{12}$.

2. How do $\frac{1}{2}$ and $\frac{2}{4}$ compare in value? Why?

How are the terms of the fraction $\frac{2}{4}$ obtained from the terms of the fraction $\frac{1}{2}$?

3. How do $\frac{2}{3}$ and $\frac{6}{9}$ compare in value? Why?

How are the terms of the fraction $\frac{6}{9}$ obtained from the terms of the fraction $\frac{2}{3}$?

4. How do $\frac{9}{12}$ and $\frac{3}{4}$ compare in value? Why?

How are the terms of the fraction $\frac{3}{4}$ obtained from the terms of the fraction $\frac{9}{12}$?

5. How do $\frac{10}{12}$ and $\frac{5}{6}$ compare in value? Why?

How are the terms of the fraction $\frac{5}{6}$ obtained from the terms of the fraction $\frac{10}{12}$?

From the examples in Questions 2–5 it is seen that:

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

When one fraction is equal in value to another, the two fractions are said to be **equivalent.**

The fraction $\frac{2}{3}$ is equal in value to the fraction $\frac{6}{9}$. Hence $\frac{2}{3}$ and $\frac{6}{9}$ are *equivalent fractions.*

When the numerator and the denominator of one fraction are greater than those of its equivalent, the first fraction is said to be in **higher terms** than the second, and the second is said to be in **lower terms** than the first.

The fraction $\frac{4}{8}$ has higher terms than its equivalent $\frac{1}{2}$. Hence $\frac{4}{8}$ is said to be the fraction of *higher terms* and $\frac{1}{2}$ the fraction of *lower terms*.

The change of $\frac{1}{4}$ to the equal fraction $\frac{3}{12}$ is called *changing* or *reducing* $\frac{1}{4}$ to higher terms.

The change of $\frac{6}{12}$ to $\frac{3}{6}$ is called reducing $\frac{6}{12}$ to *lower terms*, and the change of $\frac{6}{12}$ to $\frac{1}{2}$ is called reducing $\frac{6}{12}$ to its *lowest terms*.

A fraction is in its lowest terms when the numerator and the denominator of it are prime to each other, that is, have no common factor, for example, $\frac{3}{4}$, $\frac{7}{9}$, $\frac{15}{23}$, $\frac{6}{85}$.

Change the following fractions to their lowest terms:

1. $\frac{36}{64}$	10. $\frac{100}{225}$	19. $\frac{99}{132}$
2. $\frac{15}{36}$	11. $\frac{120}{144}$	20. $\frac{144}{156}$
3. $\frac{14}{42}$	12. $\frac{90}{135}$	21. $\frac{216}{264}$
4. $\frac{55}{155}$	13. $\frac{75}{115}$	22. $\frac{72}{108}$
5. $\frac{105}{120}$	14. $\frac{108}{132}$	23. $\frac{121}{132}$
6. $\frac{88}{96}$	15. $\frac{105}{350}$	24. $\frac{156}{188}$
7. $\frac{120}{160}$	16. $\frac{320}{400}$	25. $\frac{175}{225}$
8. $\frac{81}{144}$	17. $\frac{102}{120}$	26. $\frac{180}{540}$
9. $\frac{60}{144}$	18. $\frac{144}{128}$	27. $\frac{65}{160}$
		28. $\frac{84}{98}$

LEAST COMMON DENOMINATOR OF FRACTIONS

Fractions whose denominators are alike have a **common denominator.** For example, 12 is a common denominator of:
$$\frac{5}{12}, \frac{8}{12}, \text{ and } \frac{11}{12}.$$

A *common denominator* of two or more fractions is a number which contains the denominator of each of the fractions an integral number of times.

Thus 12, 24, and 36 may be made common denominators of the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

The **least common denominator** (L. C. D.) of two or more fractions is the *least* number which contains the denominator of each of the fractions an integral number of times.

Thus 12 is the least common denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

To find the L. C. D. of a number of fractions we simply find the L. C. M. of the denominators of these fractions. For example:

The L. C. D. of the fractions, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{7}{10}$ is the L. C. M. of 3, 5, and 10, which is 30.

The fractions $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{7}{10}$, expressed as fractions with the least common denominator will become $\frac{20}{30}$, $\frac{18}{30}$, and $\frac{21}{30}$.

EXERCISE 49

Change to fractions having the least common denominator:

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$
2. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$
3. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{9}$
4. $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{10}$
5. $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{12}$

6. $\frac{1}{3}$, $\frac{3}{4}$, $\frac{3}{8}$
7. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$
8. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$
9. $\frac{7}{8}$, $\frac{3}{10}$, $\frac{1}{2}$
10. $\frac{1}{2}$, $\frac{5}{6}$, $\frac{9}{12}$

11. $\frac{3}{7}$, $\frac{5}{14}$, $\frac{11}{21}$,
12. $\frac{5}{9}$, $\frac{7}{18}$, $\frac{11}{24}$,
13. $\frac{3}{20}$, $\frac{4}{25}$, $\frac{9}{50}$,
14. $\frac{2}{5}$, $\frac{7}{20}$, $\frac{41}{100}$,
15. $\frac{3}{4}$, $\frac{9}{10}$, $\frac{21}{50}$, $\frac{81}{100}$
16. $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{25}$

ADDITION AND SUBTRACTION OF FRACTIONS

In the addition and subtraction of whole numbers, only numbers which have the same *denomination* or *name* can be added together or subtracted from one another.

This is also true in the case of fractions.

Only those fractions which have the same name or denomination can be added or subtracted.

For instance, the sum of 3 fourths and 2 fourths is five fourths, but the sum of 3 fourths and 2 thirds is neither *thirds* nor *fourths*. If, however, both these fractions, $\frac{3}{4}$ and $\frac{2}{3}$, are changed to fractions of the same denomination or name (twelfths), the sum of 9 twelfths and 8 twelfths is 17 twelfths.

The least common denominator of the fractions $\frac{3}{4}$ and $\frac{2}{3}$ is 12.

$$\frac{3}{4} = \frac{9}{12}, \ \frac{2}{3} = \frac{8}{12}$$
$$\therefore \frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}.$$

Again, to find the value of $\frac{5}{6} - \frac{3}{8}$, we obtain the least common denominator of $\frac{5}{6}$ and $\frac{3}{8}$, which is 24.

$$\frac{5}{6} = \frac{20}{24}, \ \frac{3}{8} = \frac{9}{24}$$
$$\therefore \frac{20}{24} - \frac{9}{24} = \frac{11}{24}.$$

EXERCISE 50

Find the value of:

1. $\frac{1}{2} + \frac{1}{2}$ 15. $\frac{3}{4} + \frac{5}{10}$ 29. $\frac{3}{4} + \frac{2}{3}$

2. $\frac{2}{3} + \frac{2}{3}$ 16. $1\frac{3}{16} - \frac{1}{2}$ 30. $\frac{8}{9} + \frac{4}{5}$

3. $\frac{3}{4} + \frac{1}{4}$ 17. $\frac{5}{8} - \frac{7}{32}$ 31. $\frac{3}{4} - \frac{2}{3}$

4. $\frac{7}{8} - \frac{5}{8}$ 18. $\frac{2}{3} + \frac{5}{6}$ 32. $\frac{8}{7} + \frac{5}{10}$

5. $\frac{8}{9} - \frac{4}{9}$ 19. $\frac{2}{3} - \frac{2}{5}$ 33. $\frac{2}{7} + \frac{5}{8}$

6. $\frac{5}{10} - \frac{3}{10}$ 20. $\frac{3}{2} + \frac{5}{8}$ 34. $\frac{5}{9} + \frac{4}{5}$

7. $\frac{5}{12} + \frac{8}{12}$ 21. $\frac{4}{3} + \frac{8}{16}$ 35. $\frac{5}{6} - \frac{3}{4}$

8. $\frac{9}{20} + \frac{11}{20}$ 22. $\frac{5}{16} - \frac{8}{32}$ 36. $\frac{3}{8} + \frac{7}{10}$

9. $\frac{5}{100} + \frac{75}{100}$ 23. $\frac{7}{8} + \frac{3}{4}$ 37. $\frac{5}{8} - \frac{7}{12}$

10. $\frac{7}{10} + \frac{6}{10} + \frac{8}{10}$ 24. $\frac{3}{4} + \frac{5}{6} + \frac{7}{12}$ 38. $\frac{5}{6} + \frac{3}{10}$

11. $\frac{9}{12} + \frac{8}{12} - \frac{7}{12}$ 25. $\frac{4}{9} + \frac{5}{18} - \frac{1}{6}$ 39. $\frac{5}{6} + \frac{4}{9} - \frac{2}{3}$

12. $\frac{7}{8} - \frac{3}{8} + \frac{5}{8}$ 26. $\frac{3}{4} - \frac{1}{8} - \frac{3}{10}$ 40. $\frac{2}{5} + \frac{3}{4} - \frac{1}{2}$

13. $\frac{1}{2} + \frac{1}{4}$ 27. $\frac{2}{3} + \frac{3}{4}$ 41. $\frac{5}{8} + \frac{4}{5} - \frac{1}{4}$

14. $\frac{3}{4} - \frac{1}{2}$ 28. $\frac{1}{5} - \frac{1}{8}$

42. A grocer sold $\frac{5}{8}$ of a barrel of sugar to one customer and $\frac{3}{4}$ of a barrel to another. How many barrels did he sell altogether?

43. A farmer gave to one of his sons $\frac{1}{6}$, to another $\frac{1}{3}$, and to a third $\frac{3}{10}$ of his farm. How much of the farm did he give away? What part had he left?

44. Between Ned's house and the school there is $\frac{1}{2}$ of a mile of board walk, $\frac{1}{4}$ of a mile of cement walk, and $\frac{7}{18}$ of a mile of gravel walk. What is the total distance?

45. A boy who had \$$\frac{4}{5}$, paid \$$\frac{3}{4}$ for a book. What part of a dollar had he left?

ADDITION AND SUBTRACTION OF MIXED NUMBERS

Example: Find the value of $(4\frac{1}{5} - 2\frac{3}{4})$.

$$4\frac{1}{5} - 2\frac{3}{4} = \frac{21}{5} - \frac{11}{4}$$
$$= \frac{84}{20} - \frac{55}{20}$$
$$= \frac{29}{20} = 1\frac{9}{20}.$$

EXERCISE 51

1. Find the value of:

(a) $12 + \frac{5}{6}$	(f) $5\frac{4}{9} - 3$	(k) $5\frac{3}{4} + 2\frac{1}{6}$
(b) $3 - \frac{4}{5}$	(g) $5\frac{2}{3} + 2\frac{5}{8}$	(l) $7\frac{3}{8} - 2\frac{5}{6}$
(c) $4\frac{1}{2} + 1\frac{2}{3}$	(h) $4\frac{5}{8} - 1\frac{1}{2}$	(m) $11\frac{3}{5} + 3\frac{5}{8}$
(d) $6 - 2\frac{2}{3}$	(i) $3\frac{1}{3} - 2\frac{7}{9}$	(n) $9\frac{2}{7} - 2\frac{7}{10}$
(e) $3\frac{1}{6} + 2\frac{1}{4}$	(j) $3\frac{4}{5} - \frac{9}{10}$	(o) $15\frac{5}{8} - 9\frac{7}{12}$

2. A farmer sold two stacks of hay, in one of which there were $9\frac{3}{4}$ tons and in the other $6\frac{2}{3}$ tons. How many tons were there in the two stacks?

3. From a pole $23\frac{2}{3}$ ft. long there was cut off $4\frac{5}{8}$ ft. What was the length of the remaining part?

4. A dealer bought oats at $86\frac{3}{4}$c. a bu. and sold them at $92\frac{2}{3}$c. a bushel. What was his gain on 1 bushel?

5. A plumber worked on a job for $8\frac{5}{12}$ hours one day and for $6\frac{4}{5}$ hours the next day. How long did he work on the job?

MULTIPLICATION OF A FRACTION BY A WHOLE NUMBER

What does $\frac{2}{7} \times 3$ mean?

It means $\frac{2}{7}$ is taken 3 times, that is, $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}$.

What does $\frac{3}{8} \times 5$ mean?

It means $\frac{3}{8}$ is taken 5 times, that is,
$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{15}{8}.$$

What does $\frac{4}{5} \times 4$ mean? $\frac{6}{7} \times 8$? $\frac{8}{3} \times 7$? $\frac{4}{10} \times 6$?

Find the result in each case.

Find an easy way of multiplying a fraction by a whole number?

Example 1: Multiply $\frac{5}{6}$ by 4.
$$\frac{5}{6} \times 4 = \frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}.$$

Instead of dividing the 20 and the 6 by 2, it is found to be more convenient to divide the 4 and the 6 by 2 before multiplying.

Thus:
$$\frac{5}{\overset{}{\underset{3}{6}}} \times \overset{2}{4} = \frac{10}{3} = 3\frac{1}{3}.$$

The cancellation is not necessary in order to find the product, but it saves work, for it at once reduces the product to its lowest terms.

The product is $\frac{20}{6}$, which is equivalent to $\frac{10}{3}$.

Example 2: Multiply $2\frac{5}{8}$ by 12.
$$2\frac{5}{8} \times 12 = \frac{21}{8} \times 12 = \frac{21}{2} \times 3 = \frac{63}{2} = 31\frac{1}{2}.$$

Another solution:

$2\frac{5}{8} \times 12$ means 2 taken 12 times $+ \frac{5}{8}$ taken 12 times $=$

$$24 + \frac{5}{\underset{2}{6}} \times \overset{3}{12} = 24 + \frac{15}{2} = 24 + 7\frac{1}{2} = 31\frac{1}{2}.$$

ORAL EXERCISE

Find the value of:

1. $\frac{1}{4} \times 4$	8. $\frac{3}{10} \times 5$	15. $12 \times 2\frac{3}{4}$
2. $\frac{3}{4} \times 8$	9. $\frac{5}{7} \times 21$	16. $22 \times 6\frac{5}{11}$
3. $\frac{3}{5} \times 7$	10. $\frac{9}{25} \times 100$	17. $24 \times 3\frac{5}{8}$
4. $\frac{5}{6} \times 6$	11. $\frac{7}{8} \times 24$	18. $20 \times 5\frac{2}{5}$
5. $\frac{7}{12} \times 9$	12. $\frac{4}{9} \times 63$	19. $28 \times 3\frac{3}{4}$
6. $\frac{4}{15} \times 10$	13. $3\frac{2}{3} \times 6$	20. $2\frac{5}{12} \times 24$
7. $\frac{7}{8} \times 80$	14. $8 \times 5\frac{1}{2}$	21. $35 \times 2\frac{1}{7}$

EXERCISE 52

1. A wheat field containing 16 acres yields an average of $23\frac{3}{4}$ bu. an acre. Find the total number of bushels yielded.

2. At $68\frac{1}{4}$c. an hour what will a man earn in 2 days, working 8 hours a day?

3. A ton of coal sells for $12\frac{1}{2}$. What is the value of 15 tons?

4. At $2\frac{2}{3} a bushel what will be the value of 5 acres of wheat, if each acre yields 25 bushels?

5. A boy lives $1\frac{7}{8}$ miles from school. How far will he walk in one week—going to school in the morning and home in the evening?

6. A farmer's hens lay on an average $3\frac{1}{3}$ doz. eggs a day. How many dozen eggs will they lay in the month of April?

COMPOUND FRACTIONS
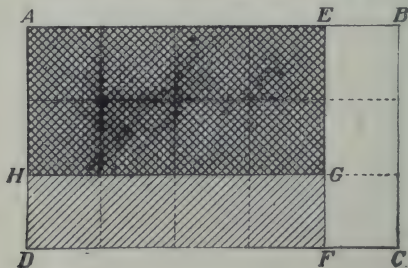
Point out $\frac{1}{5}$ of the figure ABCD.

Point out $\frac{4}{5}$ of the same figure.

Point out $\frac{1}{3}$ of the figure AEFD.

Point out $\frac{2}{3}$ of the same figure.

Then point out $\frac{2}{3}$ of $\frac{4}{5}$ of the figure ABCD.

Into how many equal parts is the figure ABCD divided?

How many of these equal parts are there in the figure AEGH?

What fraction of the figure ABCD is the figure AEGH?

To what single fraction is $\frac{2}{3}$ of $\frac{4}{5}$ equal?

How is the numerator of the single fraction related to the numerators of the two fractions $\frac{2}{3}$ and $\frac{4}{5}$?

How is the denominator of the single fraction related to the denominators of the two fractions $\frac{2}{3}$ and $\frac{4}{5}$?

By drawing figures similar to the above, show how to get a single fraction for $\frac{1}{2}$ of $\frac{1}{3}$, for $\frac{3}{4}$ of $\frac{5}{8}$.

How are the single fractions in each case obtained from the compound fractions?

Without using a figure, state to what single fraction $\frac{4}{7}$ of $\frac{3}{8}$ is equal.

How can a *fraction of a fraction,* that is, **a compound fraction,** be changed to a single fraction?

Example 1: Simplify $\frac{10}{21}$ of $\frac{14}{15}$.

$$\frac{10}{21} \text{ of } \frac{14}{15} = \frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{14}}}{\underset{3}{\cancel{21}} \times \underset{3}{\cancel{15}}} = \frac{4}{9}.$$

Here again the cancelling is done before multiplying, in order that the answer may be given in its lowest terms.

Example 2: Find the value of $\frac{4}{9}$ of $\frac{6}{7}$ of $\frac{5}{8}$.

$$\frac{4}{9} \text{ of } \frac{6}{7} \text{ of } \frac{5}{8} = \frac{4}{9} \text{ of } \frac{6 \times 5}{7 \times 8}$$

$$= \frac{\overset{1}{\cancel{4}} \times \overset{2}{\cancel{6}} \times 5}{\underset{3}{\cancel{9}} \times 7 \times \underset{2}{\cancel{8}}} = \frac{5}{21}.$$

It will be found more convenient to do the work thus:

$$\underset{3}{\overset{1}{\cancel{\frac{4}{9}}}} \text{ of } \frac{\overset{1}{\cancel{6}}}{7} \text{ of } \underset{\underset{1}{2}}{\overset{1}{\cancel{\frac{5}{8}}}} = \frac{5}{21}.$$

By what common factors have the numerator and the denominator of this compound fraction been divided?

When mixed numbers occur in examples on compound fractions, what change must first be made in the mixed numbers before cancellation and multiplication can take place?

ORAL EXERCISE

Find the value of:

1. $\frac{1}{2}$ of $\frac{1}{3}$	9. $\frac{8}{10}$ of $\frac{9}{11}$	17. $\frac{3}{4}$ of $\frac{16}{17}$
2. $\frac{1}{5}$ of $\frac{1}{6}$	10. $\frac{1}{12}$ of $\frac{6}{15}$	18. $\frac{4}{5}$ of $\frac{15}{16}$
3. $\frac{1}{4}$ of $\frac{1}{6}$	11. $\frac{7}{10}$ of $\frac{5}{7}$	19. $\frac{6}{7}$ of $\frac{14}{19}$
4. $\frac{1}{3}$ of $\frac{6}{7}$	12. $\frac{2}{3}$ of $\frac{3}{9}$	20. $\frac{1}{3}$ of $2\frac{1}{4}$
5. $\frac{1}{8}$ of $\frac{24}{25}$	13. $\frac{7}{9}$ of 27	21. $\frac{1}{8}$ of $5\frac{1}{3}$
6. $\frac{1}{10}$ of $\frac{80}{91}$	14. $\frac{3}{25}$ of 40	22. $\frac{1}{6}$ of $2\frac{2}{5}$
7. $\frac{3}{5}$ of $\frac{4}{7}$	15. $\frac{11}{20}$ of 36	23. $\frac{1}{5}$ of $7\frac{1}{2}$
8. $\frac{7}{8}$ of $\frac{5}{9}$	16. $\frac{9}{14}$ of $3\frac{1}{2}$	24. $\frac{1}{7}$ of $2\frac{3}{8}$

EXERCISE 53

1. How many pints in $\frac{5}{16}$ of a bushel?

2. How many minutes in $\frac{5}{24}$ of a day?

3. How many pounds in $\frac{2}{5}$ of a ton?

4. How many yards in $\frac{3}{4}$ of a mile?

5. A house was sold for $1850; what was $\frac{2}{3}$ of the selling price?

6. I had $45 and I spent $\frac{5}{9}$ of it for a watch; how much money had I left?

7. A lady spends $\frac{3}{5}$ of her income for board and $\frac{1}{3}$ of the remainder for clothes. What fraction of her income does she spend in clothes?

8. Of a 45-mile journey, $\frac{3}{9}$ is by rail, $\frac{3}{4}$ of the remainder by boat, and the rest by coach. What is the distance travelled by rail, by boat, and by coach?

DIVISION OF A FRACTION BY A WHOLE NUMBER

1. If the number 12 is divided by 3, what is the quotient?

What fraction of the number 12 is the quotient?
It is $\frac{1}{3}$ of 12.

2. If the number 15 is divided by 5, what is the quotient?

What fraction of the number 15 is the quotient?
It is $\frac{1}{5}$ of 15.

3. If a number is divided by 6, what fraction of the number will the quotient be? If the number is divided by 7, what fraction of the number will the quotient be? If divided by 8, by 9, by 10, by 11, in each case what fraction of the number will the quotient be?

4. If the number or dividend is $\frac{3}{4}$ and the divisor is 5, what fraction of $\frac{3}{4}$ will the quotient be?

$$\text{It will be } \tfrac{1}{5} \text{ of } \tfrac{3}{4} = \tfrac{3}{20}.$$

5. If the dividend is $\frac{4}{5}$ and the divisor 7, what fraction of $\frac{4}{5}$ will the quotient be?

$$\text{It will be } \tfrac{1}{7} \text{ of } \tfrac{4}{5} = \tfrac{4}{35}.$$

Example: $\frac{5}{7} \div 4 = \frac{1}{4}$ of $\frac{5}{7} = \frac{5}{28}$.

EXERCISE 54

1. Divide:

(a)	$\frac{2}{3}\frac{4}{5}$ by 18	(k)	$2\frac{2}{2}\frac{3}{5}$ by 48
(b)	$1\frac{5}{2}\frac{6}{5}$ by 12	(l)	$5\frac{1}{2}\frac{3}{5}$ by 23
(c)	$8\frac{3}{9}$ by 20	(m)	$28\frac{4}{7}$ by 25
(d)	$9\frac{1}{11}$ by 40	(n)	$32\frac{8}{11}$ by 21
(e)	$7\frac{2}{5}\frac{3}{6}$ by 36	(o)	$18\frac{7}{5}$ by 26
(f)	$\frac{1}{3}\frac{7}{0}$ by 51	(p)	$19\frac{1}{11}$ by 45
(g)	$1\frac{7}{2}\frac{7}{5}$ by 68	(q)	$29\frac{2}{5}$ by 42
(h)	$\frac{1}{1}\frac{9}{7}$ by 48	(r)	$19\frac{1}{2}$ by 26
(i)	$\frac{3}{5}\frac{1}{0}$ by 62	(s)	$28\frac{1}{8}$ by 18
(j)	$10\frac{2}{13}$ by 24	(t)	$5\frac{1}{2}\frac{3}{5}$ by 46

2. Divide the sum of $\frac{3}{8}$ and $\frac{7}{12}$ by 10.

3. Divide the difference between $\frac{3}{8}$ and $\frac{7}{12}$ by 10.

4. A farmer owns $\frac{3}{4}$ of a section of land and divides it into 5 farms of equal size. How many acres in each farm? (A section of land contains 640 acres.)

5. Find the average of the following sums of money:

$$\$3\tfrac{4}{5},\ \$8\tfrac{3}{10},\ \$7\tfrac{3}{4},\ \$9\tfrac{2}{3}.$$

6. If 8 yd. of cloth cost $\$2\frac{4}{5}$, what is the price of 1 yd.?

MULTIPLICATION OF A FRACTION BY A FRACTION

6 times $\frac{5}{6} = 5$.

∴ 5 is 6 times $\frac{5}{6}$.

7 times $\frac{6}{7}$ is 6.

∴ 6 is 7 times $\frac{6}{7}$.

8 times $\frac{3}{8}$ is 3.

∴ 3 is 8 times $\frac{3}{8}$.

Compare in value 4 and $\frac{4}{5}$, 3 and $\frac{3}{4}$, 6 and $\frac{6}{11}$, 5 and $\frac{5}{6}$, 7 and $\frac{7}{12}$, 9 and $\frac{9}{13}$.

Multiply 8 by 3, the product is 24.

Multiply 8 by 9, the product is 72.

In these two problems how do the multiplicands compare?

How does the multiplier 9 compare in value with the multiplier 3?

How does the product 72 compare in value with the product 24?

Provided that the multiplicand is not changed, but the multiplier is multiplied by 3, how is the product changed?

If the multiplier is made 3 times what it was, then the product becomes 3 times what it was.

If a product is 3 times what it should be, how can the product be corrected?

If the multiplier is made 5 times what it was, how is the product changed?

If the multiplier is made 7 times what it was, how is the product changed?

If 9 times, if 10 times, if 12 times, how in each case is the product changed? How can the product in each case be corrected?

8

Example 1:

Multiply $\frac{2}{5}$ by $\frac{2}{3}$.

$$\frac{2}{5} \times 2 = \frac{4}{5}.$$

The multiplier 2 has been used instead of the multiplier $\frac{2}{3}$.

Now 2 is 3 times $\frac{2}{3}$.

$\therefore$ the multiplier 2 is 3 times the multiplier that should have been used.

Hence the product $\frac{4}{5}$ is 3 times the correct product.

$\therefore$ The corrected product will be $\frac{4}{5} \div 3$.

$$\frac{4}{5} \div 3 = \frac{1}{3} \text{ of } \frac{4}{5} = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$$
$$\therefore \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}.$$

How is the numerator 4 obtained from the numerators of the two fractions? How is the denominator 15 obtained?

Example 2:

Multiply $\frac{5}{12}$ by $\frac{3}{10}$.

$$\frac{5}{12} \times \frac{3}{10} = \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{10}}} = \frac{1}{8}.$$

Example 3:

$$2\tfrac{1}{3} \times 3\tfrac{1}{9} \times 4\tfrac{2}{7}.$$

The mixed numbers being brought to improper fractions, the question becomes:

$$\frac{7}{3} \times \frac{28}{9} \times \frac{30}{7} = \frac{7}{3} \times \frac{\overset{4}{\cancel{28}}}{9} \times \frac{\overset{10}{\cancel{30}}}{\underset{1}{\cancel{7}}} = \frac{7 \times 4 \times 10}{9}$$

$$= \frac{280}{9} = 31\tfrac{1}{9}.$$

EXERCISE 55

1. Find the value of each of the following in its lowest terms:

(a) $\frac{1}{25} \times \frac{10}{21}$ (k) $8\frac{2}{3} \times 4\frac{1}{2}$

(b) $\frac{13}{20} \times \frac{15}{26}$ (l) $2\frac{4}{7} \times \frac{5}{9} \times \frac{21}{40}$

(c) $\frac{8}{15} \times \frac{25}{86}$ (m) $6\frac{2}{5} \times \frac{25}{42} \times 2\frac{5}{8}$

(d) $\frac{27}{32} \times \frac{44}{45}$ (n) $2\frac{1}{3} \times 3\frac{1}{9} \times 4\frac{2}{7}$

(e) $\frac{18}{35} \times \frac{14}{27}$ (o) $2\frac{5}{6} \times \frac{4}{15} \times 1\frac{11}{34}$

(f) $2\frac{1}{5} \times \frac{15}{22}$ (p) $5\frac{1}{8} \times 4\frac{3}{4} \times 8\frac{1}{2} \times \frac{1}{9}$

(g) $6\frac{2}{3} \times \frac{9}{10}$ (q) $4\frac{2}{3} \times \frac{2}{7} \times \frac{3}{8} \times \frac{3}{5}$

(h) $\frac{16}{21} \times 8\frac{3}{4}$ (r) $3\frac{1}{3} \times 4\frac{1}{5} \times \frac{7}{10}$

(i) $5\frac{1}{3} \times 2\frac{4}{7}$ (s) $4\frac{23}{25} \times \frac{10}{41}$ of $2\frac{32}{33}$

(j) $9\frac{2}{7} \times 3\frac{3}{13}$ (t) $\frac{5}{7}$ of $1\frac{1}{12} \times \frac{9}{14}$ of $6\frac{1}{8}$.

2. At $\$\frac{4}{5}$ a yd. what will $15\frac{5}{8}$ yd. of cloth cost?

3. At $\$10\frac{1}{2}$ a cwt., what will $2\frac{1}{2}$ cwt. of flour cost?

4. If a train runs at the rate of $22\frac{1}{2}$ miles an hour, how far will it run in $8\frac{1}{4}$ hours?

5. Find the cost of:

(a) $12\frac{1}{5}$ bu. of potatoes @ $\$1\frac{3}{4}$ a bu.

(b) $25\frac{1}{4}$ gal. of milk @ $37\frac{1}{2}$c. a gal.

(c) $36\frac{3}{4}$ lb. of nails @ $27\frac{1}{2}$c. a lb.

(d) $8\frac{1}{3}$ lb. of butter @ $60\frac{1}{2}$c. a lb.

DIVISION OF A FRACTION BY A FRACTION

3 is how many times $\frac{3}{4}$?

4   "   "   " $\frac{4}{5}$?

6   "   "   " $\frac{6}{7}$?

Compare in value $\frac{2}{3}$ and 2; $\frac{1}{4}$ and 1; $\frac{5}{8}$ and 5; $\frac{7}{8}$ and 7; $\frac{9}{10}$ and 9.

Divide 48 by 3.   Quotient is 16.

Divide 48 by 12.   Quotient is 4.

How do the dividends compare?

How does the divisor 12 compare in value with the divisor 3?

How does the quotient 4 compare in value with the quotient 16?

Provided the dividend is not changed, but the divisor is multiplied by 4, how is the quotient changed?

If the divisor is made 4 times what it was, then the quotient becomes $\frac{1}{4}$ of what it was.

If the quotient is $\frac{1}{4}$ of what it should be, how can the quotient be corrected?

<center>ORAL EXERCISE</center>

1. Divide 45 by 3.
   Divide 45 by 15.
   If the divisor is multiplied by 5, how is the quotient changed?
   How can the quotient be corrected?

2. Divide 96 by 8.
   Divide 96 by 16.
   If the divisor is multiplied by 2, how is the quotient changed?
   How can the quotient be corrected?

3. Divide 120 by 5.
   Divide 120 by 40.
   If the divisor is multiplied by 8, how is the quotient changed?
   How can the quotient be corrected?

Example 1:

Divide $\frac{4}{5}$ by $\frac{3}{4}$.

$$\frac{4}{5} \div 3 = \frac{1}{3} \text{ of } \frac{4}{5} = \frac{4}{15}.$$

The divisor 3 has been used instead of the correct divisor $\frac{3}{4}$.

$$3 \text{ is } 4 \text{ times } \tfrac{3}{4}.$$

$\therefore$ A divisor has been used which is 4 times the correct divisor $\frac{3}{4}$.

$\therefore$ The quotient $\frac{4}{15}$ is only $\frac{1}{4}$ of what the quotient should be,

$\therefore$ The correct quotient is 4 times $\frac{4}{15} = \frac{16}{15}$.

$$\frac{16}{15} = \frac{4}{5} \times \frac{4}{3}.$$
$$\text{Hence } \frac{4}{5} \div \frac{3}{4} = \frac{4}{5} \times \frac{4}{3}.$$

That is, to divide one fraction by another fraction, *invert the fraction which is the divisor and then use it as a multiplier.*

Another method:

Divide $\frac{4}{5}$ by $\frac{3}{8}$.

$$\begin{aligned}
\frac{4}{5} \div \frac{3}{8} &= \frac{32}{40} \div \frac{15}{40} \\
&= 32 \text{ fortieths} \div 15 \text{ fortieths} \\
&= 32 \div 15 \\
&= \frac{32}{15} \\
&= \frac{4}{5} \times \frac{8}{3}.
\end{aligned}$$

Hence the rule.

Example 2:

Divide $\frac{5}{9}$ by $\frac{10}{6}$.

$$\frac{5}{9} \div \frac{10}{6} = \frac{5}{9} \times \frac{6}{10} = \frac{\cancel{5}^{1}}{\cancel{9}_{3}} \times \frac{\cancel{6}^{\cancel{2}^{1}}}{\cancel{10}_{\cancel{2}_{1}}} = \frac{1}{3}.$$

Example 3:

Divide 8 by $\frac{3}{7}$.

$$8 \div \frac{3}{7} = 8 \times \frac{7}{3} = \frac{56}{3} = 18\frac{2}{3}.$$

<center>EXERCISE 56</center>

Find the value of:

1. $\frac{1}{2} \div \frac{1}{2}$	11. $\frac{7}{15} \div \frac{9}{10}$	21. $9 \div \frac{4}{5}$
2. $\frac{1}{2} \div \frac{1}{4}$	12. $2\frac{1}{4} \div \frac{8}{9}$	22. $12 \div \frac{3}{4}$
3. $\frac{1}{2} \div \frac{3}{4}$	13. $2\frac{3}{8} \div \frac{4}{15}$	23. $8 \div \frac{3}{10}$
4. $\frac{1}{3} \div \frac{1}{2}$	14. $3\frac{1}{7} \div 2\frac{1}{5}$	24. $16 \div \frac{8}{9}$
5. $\frac{1}{3} \div \frac{1}{4}$	15. $5\frac{3}{8} \div 7\frac{1}{6}$	25. $8\frac{2}{3} \div 2\frac{1}{6}$
6. $\frac{5}{8} \div \frac{1}{2}$	16. $6\frac{1}{4} \div 2\frac{1}{2}$	26. $62\frac{1}{2} \div 6\frac{1}{4}$
7. $\frac{4}{9} \div \frac{2}{3}$	17. $7\frac{1}{7} \div 4\frac{1}{6}$	27. $\frac{9}{24} \div \frac{5}{16}$
8. $\frac{5}{8} \div \frac{5}{6}$	18. $1\frac{4}{15} \div 1\frac{8}{11}$	28. $1\frac{4}{15} \div 1\frac{1}{12}$
9. $1\frac{1}{24} \div \frac{7}{8}$	19. $4\frac{3}{8} \div 9\frac{2}{7}$	29. $\frac{17}{52} \div \frac{51}{52}$
10. $\frac{9}{16} \div \frac{5}{12}$	20. $6 \div \frac{3}{5}$	30. $5\frac{3}{8} \div (\frac{7}{9} \text{ of } \frac{5}{21})$

31. At $\$\frac{3}{8}$ a yard, how many yards of ribbon can be purchased for $\$7\frac{1}{8}$?

32. If for $\frac{3}{4}$ of an acre of land there is paid $\$37\frac{1}{2}$, what is the price of one acre?

33. At the rate of $3\frac{1}{3}$ miles an hour, how long will it take a man to walk $11\frac{2}{3}$ miles?

34. How many bags will be required for $31\frac{1}{2}$ bu. of grain if each bag will hold $2\frac{1}{4}$ bu.?

<center>COMPARISON OF QUANTITIES</center>

What fraction of $6 is $2?

The fraction of $6 to which $2 is equal is $\frac{\$2}{\$6} = \frac{2}{6} = \frac{1}{3}$.

Proof—　　　$\frac{1}{3}$ of $6 = $2.

What fraction of $8 is $5?

The fraction of $8 to which $5 is equal is $\frac{\$5}{\$8} = \frac{5}{8}$.

Proof—　　　$\frac{5}{8}$ of $8 = $5.

What fraction of $\frac{4}{5}$ is $\frac{2}{3}$?

The fraction of $\frac{4}{5}$ to which $\frac{2}{3}$ is equal is $\frac{\frac{2}{3}}{\frac{4}{5}}$.

$$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{\overset{}{2}9}{3} \times \frac{5}{\underset{2}{4}} = \frac{5}{6}.$$

Proof—
$$\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} \text{ of } \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} = \tfrac{2}{3}.$$

What fraction of $3\frac{1}{3}$ is $2\frac{1}{2}$?

The fraction of $3\frac{1}{3}$ to which $2\frac{1}{2}$ is equal is:

$$\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{\frac{5}{2}}{\frac{10}{3}} = \tfrac{5}{2} \div \tfrac{10}{3} = \tfrac{5}{2} \times \tfrac{3}{10} = \tfrac{3}{4}.$$

Proof—$\tfrac{3}{4}$ of $3\frac{1}{3} = \tfrac{3}{4}$ of $\tfrac{10}{3} = \tfrac{5}{2} = 2\frac{1}{2}.$

EXERCISE 57

1. What fraction of $1\frac{1}{2}$ ft. is 7 in.?
2. "   "   " 3 dy. is 8 hr.?
3. "   "   " 2 wk. is 3 da. 12 hr.?
4. "   "   " £3 is 10s. 6d.?
5. "   "   " 6 ft. 3 in. is 2 ft. 9 in.?
6. "   "   " 10 lb. 12 oz. is 4 lb. 8 oz.?

7. Express:

    (a) $\tfrac{5}{6}$ of 8 yd. as a fraction of $\tfrac{3}{8}$ of 4 rd.

    (b) $\tfrac{5}{8}$ of 9s. 4d. as a fraction of $\tfrac{2}{3}$ of £4 10s.

    (c) $\tfrac{2}{3}$ of 1500 lb. as a fraction of $\tfrac{3}{4}$ of 2 tons, 8 cwt.

    (d) $\tfrac{4}{7}$ as the fraction of $\tfrac{1}{2}$.

    (e) $\tfrac{7}{12}$ as the fraction of $\tfrac{3}{8}$.

    (f) $1\frac{1}{2}$ as the fraction of $2\frac{1}{2}$.

    (g) $4\frac{5}{7}$ as the fraction of $6\frac{2}{7}$.

COMPLEX FRACTIONS

Express $4 \div 5$ as a single fraction.   Ans. $\tfrac{4}{5}$.

    " $3 \div \tfrac{4}{5}$ "   "   "   " $\dfrac{3}{\frac{4}{5}}$

    " $\tfrac{2}{7} \div 4$ "   "   "   " $\dfrac{\frac{2}{7}}{4}$

    " $5\frac{1}{3} \div 2\frac{1}{5}$ as a single fraction.   Ans. $\dfrac{5\frac{1}{3}}{2\frac{1}{5}}.$

A fraction such as $\frac{4}{5}$, whose numerator and denominator are whole numbers, is called a **simple fraction.**

Fractions such as $\frac{3}{\frac{2}{5}}$, $\frac{\frac{2}{7}}{4}$, and $\frac{5\frac{1}{3}}{2\frac{1}{9}}$ are called **complex fractions.**

A complex fraction is a fraction which has a fraction in its numerator or in its denominator or in both.

EXERCISE 58

Simplify:

1. $\dfrac{6\frac{2}{5}}{8}$

2. $\dfrac{7\frac{1}{7}}{25}$

3. $\dfrac{8\frac{3}{4}}{70}$

4. $\dfrac{9}{4\frac{1}{2}}$

5. $\dfrac{18}{7\frac{1}{5}}$

6. $\dfrac{64}{9\frac{5}{11}}$

7. $\dfrac{2\frac{1}{3}}{4\frac{1}{5}}$

8. $\dfrac{8\frac{1}{8}}{5\frac{1}{5}}$

9. $\dfrac{4\frac{1}{11}}{7\frac{7}{8}}$

10. $\dfrac{\frac{2}{3} \text{ of } 1\frac{5}{6}}{1\frac{1}{2} - \frac{3}{8}}$

11. $\dfrac{3\frac{1}{7} \times \frac{28}{33}}{1\frac{5}{6} - \frac{1}{2}}$

12. $\dfrac{\frac{1}{2} + \frac{3}{4}}{\frac{3}{4} - \frac{1}{2}}$

13. $1\frac{1}{2}$ of $\frac{8}{9} + \frac{2}{5}$ of $3\frac{3}{4}$

14. $3\frac{3}{4} - \frac{2}{5}$ of $2\frac{1}{2}$.

EXERCISE 59 (REVIEW)

1. Reduce to their lowest terms:

(a) $\frac{39}{51}$ (b) $\frac{57}{190}$ (c) $\frac{117}{273}$ (d) $\frac{69}{230}$ (e) $\frac{84}{144}$ (f) $\frac{195}{408}$
    (g) $\frac{210}{336}$.

2. Reduce:

(a) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{12}$ to sixtieths

(b) $\frac{3}{7}$, $\frac{4}{12}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{9}{14}$, $\frac{11}{42}$ to eighty-fourths

(c) $\frac{5}{10}$, $\frac{4}{20}$, $\frac{4}{5}$, $\frac{7}{50}$, $\frac{8}{25}$ to hundredths.

3. Reduce these fractions to equivalent fractions having the lowest common denominator:

(a) $\frac{2}{3}$, $\frac{5}{9}$, $\frac{3}{4}$, $\frac{7}{6}$

(b) $\frac{3}{7}$, $\frac{11}{21}$, $\frac{9}{14}$, $\frac{2}{3}$

(c) $\frac{1}{3}$, $\frac{31}{4}$, $4\frac{1}{5}$, $\frac{3}{20}$, $2\frac{7}{15}$.

4. Add:

(a) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{12}$

(b) $\frac{4}{5}$, $\frac{7}{10}$, $\frac{9}{20}$, $\frac{11}{30}$

(c) $1\frac{3}{5}$, $2\frac{3}{8}$, $3\frac{3}{10}$, $4\frac{9}{50}$.

5. Subtract:

(a) $\frac{3}{4}$ from $\frac{7}{8}$

(b) $\frac{8}{9}$ from $1\frac{1}{2}$

(c) $4\frac{3}{4}$ from $7\frac{9}{10}$

(d) $5\frac{8}{9}$ from $7\frac{5}{18}$.

6. Find by the method of cancellation the value of the following, each in its lowest terms:

(a) $\dfrac{75 \times 63 \times 27}{100 \times 48 \times 72}$

(b) $\dfrac{42 \times 57 \times 26}{21 \times 91 \times 19}$

(c) $\dfrac{32 \times 38 \times 39 \times 45}{80 \times 95 \times 91 \times 60}$.

7. Find the value of:

(a) $\frac{5}{6}$ of $\frac{2}{7}$

(b) $\frac{5}{8}$ of $\frac{4}{9}$

(c) $\frac{15}{21}$ of $3\frac{8}{13}$

(d) $\frac{11}{76}$ of $1\frac{19}{22}$.

8. Multiply:

(a) $\frac{1}{2} \times \frac{2}{3}$	(d) $4\frac{2}{3} \times 9$	(g) $8\frac{2}{3} \times 9\frac{3}{4}$
(b) $\frac{3}{4} \times \frac{3}{4}$	(e) $8\frac{3}{4} \times 12$	(h) $9\frac{3}{4} \times 12\frac{2}{3}$
(c) $\frac{3}{9} \times \frac{5}{8}$	(f) $20\frac{5}{6} \times 18$	(i) $15\frac{4}{5} \times 10\frac{2}{5}$.

9. Divide:

    (a) $1\frac{1}{3}$ by 6   (d) 12 by $\frac{3}{4}$ (g) $\frac{5}{8}$ by $\frac{1}{4}$

    (b) $\frac{6}{8}$ by 3   (e) 15 by $\frac{2}{3}$ (h) $\frac{1}{7}$ by $\frac{5}{8}$

    (c) $\frac{9}{10}$ by 4   (f) 10 by $\frac{1}{2}$ (i) $3\frac{1}{3}$ by $4\frac{4}{7}$.

10. How many ribbons $\frac{2}{3}$ of a yard long can be cut from $46\frac{2}{3}$ yd. of ribbon?

11. An automobile ran $63\frac{1}{3}$ miles in $3\frac{1}{8}$ hours. Find the average rate per hour.

12. James weighs $160\frac{3}{4}$ lb., Sarah 108 lb., John $135\frac{3}{4}$ lb., Mary $120\frac{5}{8}$ lb., and Henry $124\frac{7}{8}$ lb. Find their average weight.

13. $\frac{4}{5}$ of the length of a flagpole is 60 ft. What is the length of $\frac{2}{3}$ of the pole?

14. A man who spent $\frac{7}{10}$ of his money found that he had $17\frac{1}{2}$ left. How much had he at first?

15. A farmer sells $\frac{2}{5}$ of his farm and gives his son $\frac{1}{2}$ of the remainder. In what he now has there are 75 acres. How many acres were there in the farm at first?

16. A mixture contains $33\frac{1}{3}$ lb. of corn, $25\frac{5}{8}$ lb. of oats, and $17\frac{3}{4}$ lb. of barley. How many cows will it feed if $5\frac{1}{8}$ lb. is given to each cow?

17. From $232\frac{1}{2}$ yd. of cotton, $185\frac{2}{3}$ yd. were sold. What will the remainder be worth at 24c. a yd.?

18. Make out a bill for the following:

        555 lb. wheat @ $2·80 a bu.

        289 lb. oats @ $1·10 a bu.

        432 lb. peas @ $3·30 a bu.

19. Make out a bill for the following:

        288 lb. timothy seed @ 8\frac{1}{2}$ a bu.

        444 lb. barley @ 88c. a bu.

        1560 lb. hay @ $25 a ton.

LINEAR MEASURE

The following table has already been learned:

$$12 \text{ in.} = 1 \text{ ft.}$$
$$3 \text{ ft.} = 1 \text{ yd.}$$
$$1760 \text{ yd.} = 1 \text{ mile.}$$

There is another unit of length longer than the yard, which is very often used to measure the length of fields, and distances which may be less than one mile. This unit is called the **rod.**

If there are 320 rods in 1 mile, find the number of yd. in one rod.

Re-write the table of linear measure including the new unit, the rod.

EXERCISE 60

1. How many rods in 2 miles? in 3 miles? in $2\frac{1}{2}$ miles? in $3\frac{3}{4}$ miles? in 4 miles, 110 rods?

2. How many miles in 640 rods? in 160 rods? in 40 rods? in 1280 rods? in 2240 rods?

3. How many miles and rods in 1400 rods? in 1560 rods? in 1760 rods?

4. If a farm is 80 rods wide, how many farms of the same width will it take to make a distance of 5 miles?

5. From one concession to another is $1\frac{1}{4}$ miles. How many rods in that distance?

6. A field is $\frac{1}{8}$ of a mile long. How much will it cost to build a fence along the field at $1·75 a rod?

7. How many yd. in 2 rods? in 6 rods? in 14 rods? in 11 rods? in 15 rods?

8. How many rods in 22 yd.? in 33 yd.? in 55 yd.? in 88 yd.?

9. How many rods and yd. are there in 8 yd.? in 12 yd.? in 19 yd.? in 27 yd.?

10. What will it cost to pave a street 160 rods long at $12·25 a yd.?

11. A ditch 77 yd. long cost $2·25 a rod. What was the cost of the ditch?

12. How many yd. in 2 miles, 67 rods, 4 yd.?

SCALE-DRAWING—PERIMETER

If a distance of one foot is represented in a drawing by a line one inch in length, the scale used is said to be 1 in. to the foot.

If the distance represented by one inch is one yard, the scale used is 1 in. to the yard.

If a distance of 10 feet is represented in a drawing by a line ½ inch in length, the scale used is ½ in. to 10 feet.

EXERCISE 61

Draw a line to represent the distance:

1. 100 ft., if the scale is 1 in. to 20 ft.

2. Draw a line to represent a distance of 50 yd., if the scale is 1 in. to 20 yd.

3. Draw a line to represent a distance of 75 rods, if the scale is 1 in. to 30 rods.

4. Draw a diagram on a scale of 1 in. to 8 ft. for a room 16 ft. long and 12 ft. wide.

What is the distance in inches around the diagram?

What is the distance in feet around the room?

5. Draw diagrams on a scale of 1 in. to 8 ft. for rooms of the dimensions given below, and find the perimeter of each room in feet:

> (a) 24 ft. long, 16 ft. wide,
> (b) 18 "   " 12 "   "
> (c) 20 "   " 14 "   "

6. Draw a diagram on a scale of 1 in. to 16 yd. for lots of the dimensions given below, and find the perimeter of each lot in yards:

> (a) 32 yd. long, 24 yd. wide
> (b) 48 "   "   36 "   "
> (c) 56 "   "   42 "   "

7. Draw diagrams on a scale of $\frac{1}{4}$ in. to 5 rods for fields of the dimensions given below, and find the perimeter of each field in rods:

> (a) 40 rods long, 40 rods wide
> (b) 80 "   "   20 "   "
> (c) 60 "   "   30 "   "

8. What will it cost to plant a hedge around a lot 12 rods long and 8 rods wide, if the plants are placed 1 foot apart and cost $4·25 per hundred?

9. The length of a rectangular lot is 100 yd., and its width is 40 yd. What will it cost to fence it with wire netting at $7·20 per hundred feet and posts set 10 ft. apart and costing 75c. each?

10. A barn is 80 ft. long and 60 ft. wide. How many boards 8 in. wide, placed upright, will it take to go around the barn?

11. A trunk is 2 ft. wide and $1\frac{1}{2}$ ft. high. What length of strap will be necessary to go around it and leave one foot of strap over?

12. A playground is 18 rods long and 16 rods wide. What will it cost to fence it at 75c. a yd.?

13. How many posts placed 10 feet apart will be required for a fence around a field 40 rods long and 40 rods wide?

14. How many posts will be required for a fence along the front and one side of the field in Question (13)?

15. What length of fence in miles is required to go around a farm 160 rods wide and 100 rods long?

MEASUREMENT OF AREA

The surface of a solid is that part of it which can be seen and touched. It is the outside of the solid. In your school-room, note that there are two kinds of surfaces—flat, or plane, surfaces, and round, or curved, surfaces. Note a few examples of each.

The size of a surface is called its **area.** This area is generally measured in units which are squares—the square inch, the square foot, the square yard, the square rod, and the square mile. The acre which is used for measuring fields and farms is not a square.

Draw (a) upon your work book a square inch.

(b) upon the black-board a square foot and a square yard.

(c) upon the floor a square rod.

Measure off an acre on a field which is 40 rods in length. Draw an acre to the scale of 1 in. to 10 rods.

Define a square mile. Draw a square mile to scale.

THE AREA OF A RECTANGLE

Draw a rectangle 4 in. long and 3 in. wide. Divide each side of the rectangle into inches. Join corresponding points on opposite sides of the rectangle.

How many squares in each row?

How many rows? How many squares altogether?

Express the number of squares as a product
$$= 4 \times 3.$$

What is the area of each square?

What is the area of the rectangle?

$4 \times 3 \times 1$ sq. in. = 12 times 1 sq. in. = 12 sq. in.

From these examples it is evident that the number of square units in the area of a rectangular space is equal to the number of linear units in its length multiplied by the number of linear units in its width. All linear units must be of the same denomination.

ORAL EXERCISE

1. Express and find the area of each of the following rectangles.

> (a) 8 in. long, 6 in. wide
> (b) 9 in. long, 8 in. wide
> (c) 15 in. long, 20 in. wide
> (d) 7 ft. long, 9 ft. wide
> (e) 6 ft. long, 8 ft. wide
> (f) 25 ft. long, 20 ft. wide
> (g) 12 yd. long, 5 yd. wide
> (h) 9 yd. long, 11 yd. wide
> (i) 20 rods long, 15 rods wide
> (j) 40 rods long, 40 rods wide
> (k) 6 mi. long, 6 mi. wide.

2. Find the number of square feet in a lot 25 ft. wide and 120 ft. deep.

3. How many square miles of land in a township which is 6 mi. long and 6 mi. wide?

SQUARE MEASURE

EXERCISE 62

A. THE SQUARE FOOT AND THE SQUARE INCH

1. Draw a square foot upon the black-board. Divide it into sq. in. Find how many sq. in. in a square foot.

2. How many sq. in. in 2 sq. ft.? in 3 sq. ft.? in 5 sq. ft.? in 2 sq. ft. 12 sq. in.? in 3 sq. ft. 18 sq. in.? in 4 sq. ft. 100 sq. in.?

3. How many sq. ft. in 288 sq. in.? in 432 sq. in.? in 864 sq. in.? in 2880 sq. in.?

4. How many sq. ft. and sq. in. in 450 sq. in.? in 382 sq. in.? in 875 sq. in.? in 900 sq. in.?

5. A table is 96 in. long and 48 in. wide. How many sq. ft. of surface has it?

6. A rug is 10 ft. 10 in. long and 8 ft. 4 in. wide. How many sq. ft. in its area?

7. A verandah is 22 ft. long and 7 ft. wide. How many sq. in. of flooring will it require?

B. THE SQUARE YARD AND THE SQUARE FOOT

1. Draw a square yard upon the black-board. Divide it into sq. ft. Find how many sq. ft. there are in a square yard?

2. How many sq. ft. in 8 sq. yd.? in 12 sq. yd.? in 15 sq. yd.? in 25 sq. yd.?

3. How many sq. ft. in 5 sq. yd. 2 sq. ft.? in 7 sq. yd. 3 sq. ft.? in 15 sq. yd. 6 sq. ft.?

4. A floor is 9 yd. long and 8 yd. wide. How many sq. ft. of floor space does it contain?

5. How many sq. ft. of glass in a window that is 3 yd. 2 ft. wide and 2 yd. 2 ft. high?

6. A black-board is 4 ft. wide and 36 ft. long. How many sq. yd. in it?

C. THE SQUARE ROD AND THE SQUARE YARD

1. Mark off on your school ground a square rod. Divide it into square yards, half square yards, and one quarter square yard. Find how many square yards = 1 sq. rod.

2. How many sq. yd. in 2 sq. rods? in 3 sq. rods? in 4 sq. rods? in 4 sq. rods, 12 sq. yd.? in 6 sq. rods, 25 sq. yd.?

3. A field is 40 rods long and 40 rods wide. What is its area in sq. yd.?

4. What are the areas in sq. yd. of the following rectangular lots?

    (a) 10 rods long, 8 rods wide
    (b) 15 "   "   6 "   "
    (c) 12 "   "   9 "   "

D. THE ACRE AND THE SQUARE ROD

1. On a field which is 40 rods long mark off a width of 4 rods. How many sq. rods in this area?

An area which contains 160 sq. rods is called an **acre**.

2. Give the dimensions of 4 dissimilar fields each of which contains an acre.

3. How many acres in a field which is 40 rods long and 40 rods wide?

4. Find the area in acres of the following rectangular fields:

    (a) 60 rods long, 80 rods wide
    (b) 20 "   "   120 "   "
    (c) 80 "   "   100 "   "
    (d) 320 "   "   320 "   "
    (e) 45 "   "   32 "   "
    (f) 72 "   "   48 "   "

E. THE SQUARE MILE, THE SECTION, AND THE ACRE

1. A rectangular area of land which is a mile long and a mile wide is called a **square mile**. In western Canada it is called a **section** of land.

2. How many acres of land are there in a square mile? in a section? in 2 sections? in $\frac{1}{2}$ section? in $\frac{3}{4}$ section? in $2\frac{1}{4}$ sections? in $3\frac{3}{4}$ sections? in $4\frac{1}{2}$ sections?

Fill in the blanks in the following table and memorize the facts:

.... sq. in. = 1 sq. ft.   .... sq. rod = 1 acre
.... sq. ft. = 1 sq. yd.   .... acres = 1 sq. mile
.... sq. yd. = 1 sq. rod   = 1 section.

EXERCISE 63

1. How many acres are there in rectangular fields of these dimensions: 38 rd. by 20 rd.? 48 rd. by 30 rd.? 60 rd. by 32 rd.? 54 rd. by 72 rd.?

2. Find the dimensions of your school grounds, and find the number of acres in them.

3. For a farm 240 rd. long and 144 rd. wide a man paid $65 an acre. What did the farm cost him?

4. A city bought a street 66 ft. wide and 180 rd. long @ $100 an acre. How much did the street cost?

5. At 23c. a sq. ft. what will be the cost of building a cement walk $\frac{1}{4}$ mi. long and $4\frac{1}{2}$ ft. wide?

6. Around the outside of a block of land 40 rd. by 20 rd. there is a cement walk 4 ft. wide. What did the walk cost @ 27c. a sq. ft.?

7. At $3\frac{1}{2}$c. a sq. ft. what will it cost to sod a strip of ground 10 rd. long and 36 ft. wide?

8. Find the width of a field which contains 2 acres and which is 20 rods long?

9. Find the cost of painting the floor of a room 24 ft. × 18 ft. @ 36c. a sq. yd.

10. Around a 2-acre lot which is 20 rd. long the owner builds a wire fence with posts 22 ft. apart. How many posts are required?

11. Find the cost of painting both sides of a tight board fence 50 ft. long and 6 ft. high @ 25c. a square yard.

12. If it requires 72 rods of fence to inclose a square field, how long is a side of the field?

How many acres does the field contain?

MEASUREMENT OF VOLUME

The **volume** of a solid is the amount of space it occupies. Volume is usually measured in units which are **cubes**, for example, the *cubic inch*, the *cubic foot*, and the *cubic yard*. The **cord**, however, which is used as a unit to measure wood and stone, is not a cube.

ORAL EXERCISE

1. Build up a rectangular solid from inch cubes. Put 4 cubes in the length, 3 cubes in the width, and 2 cubes in the height.

2. How many cubes in each layer? How many cubes altogether?

$$4 \times 3 \times 2$$

3. What is the volume of each cube? What is the volume of the rectangular solid?

$$4 \times 3 \times 2 \times 1 \text{ cu. in.} = 24 \text{ times } 1 \text{ cu. in.} = 24 \text{ cu. in.}$$

4. How many cubes in a rectangular pile which has 5 cubes in the length, 4 cubes in the width, and 3 cubes in the height?

If each cube is a cu. in., what is the volume of the solid?

$$5 \times 4 \times 3 \times 1 \text{ cu. in.}$$

5. How many cubes in a rectangular pile which has 6 cubes in the length, 5 cubes in the width, and 4 cubes in the height? If each cube is a cu. ft., find the volume of the pile.

6. How many cubes, each of which is a cu. in., are there in a rectangular pile which is 4 in. long, 3 in. wide, and 3 in. high? Find the volume of the pile.

7. How many cubes, each of which is a cu. ft., are there in a pile 6 ft. long, 4 ft. wide, and 3 ft. high? Find the volume of the pile in cu. ft.

From these examples it is evident that the number of cubic units in the volume of any rectangular solid is equal

to the *number* of linear units in its length, multiplied by the *number* of linear units in its width, multiplied by the *number* of linear units in its height.

All the linear units must be of the same denomination.

ORAL EXERCISE

1. Find the number of cubic inches in rectangular blocks whose dimensions are:

  (a) 3 in. × 2 in. × 1 in.   (c)   6 in. × 5 in. × 3 in.
  (b) 4 in. × 3 in. × 2 in.   (d) 10 in. × 8 in. × 5 in.

2. Find the number of cu. ft. in rectangular solids whose dimensions are:

  (a) 9 ft. × 5 ft. × 3 ft.   (c) 6 ft. × 24 in. × 12 in.
  (b) 8 ft. × 6 ft. × 3 ft.   (d) 4 ft. × 18 in. × 8 in.

3. Find the third dimension when the other dimensions are 4 in. and 3 in. and the volume is 36 cu. in.

4. How many cubic inches of earth can be put in a box 8 in. × 4 in. × 3 in.?

5. How many cu. yd. of earth must be removed to leave a rectangular hole 9 ft. × 3 ft. × 2 ft.?

6. How many cu. ft. of grain will a bin 5 ft. wide, 8 ft. long, and 7 ft. high hold?

7. How many cu. ft. in a pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high?

CUBIC MEASURE

THE CUBIC INCH AND THE CUBIC FOOT

1. Find the number of cu. in. in a rectangular pile 12 in. long, 12 in. wide, and 12 in. high.

2. Express the dimensions of this pile in feet.   Find its volume in cu. ft.

How many cu. in. in 1 cu. ft.?

EXERCISE 64

1. How many cu. in. in 2 cu. ft.? in 3 cu. ft.? in 4 cu. ft.? in 2 cu. ft. 36 cu. in.? in 5 cu. ft. 72 cu. in.?

2. A solid contains 10368 cu. in. How many cu. ft. does it contain?

3. A tank holds 31104 cu. in. of water. How many cu. ft. of water will it hold?

4. A block of cement contains 33654 cu. in. How many cu. ft. and cu. in. does it contain?

THE CUBIC FOOT AND THE CUBIC YARD

1. A rectangular solid is 3 ft. long, 3 ft. wide, and 3 ft. high. How many cu. ft. does it contain?

2. Express the dimensions of this solid in yards. What is its volume in cu. yd.?

How many cu. ft. in 1 cu. yd.?

EXERCISE 65

1. How many cu. ft. in 2 cu. yd.? in 3 cu. yd. 18 cu. ft.? in 5 cu. yd. 16 cu. ft.? in 7 cu. yd. 11 cu. ft.?

2. How many cu. yd. must be removed in digging a cellar 36 ft. long, 24 ft. wide, and 7 ft. deep?

3. In digging a drain, 3429 cu. yd. of earth were taken out. How many cu. ft. of earth were removed?

4. How many cu. yd. of gravel will be required to gravel a road 1890 yd. long, 3 ft. wide, and 1 ft. deep?

5. If a load of gravel contains 1 cu. yd., how many loads of gravel will be required in Question (4)? What will the gravel cost at 75c. a load?

THE CUBIC FOOT AND THE CORD

1. How many cu. ft. of wood are there in a pile 8 ft. long, 4 ft. wide, and 4 ft. high?

A pile of wood or stone containing *128 cu. ft.* is said to contain 1 cord.

EXERCISE 66

How many cords in each of the following piles of wood? Use the cancellation method.

1. 12 ft. long,   8 ft. wide, 6 ft. high
2. 16 "   "   12 "   "   8 "   "
3. 24 "   "   16 "   "   4 "   "
4. 72 "   "   12 "   "   6 "   "
5. 96 "   "   24 "   "   8 "   "
6. 84 "   "   8 "   "   5 "   "
7. 108 "   "   12 "   "   7 "   "

8. Find the value of the piles in each of Questions 5, 6, and 7 if wood is worth $15 a cord.

Fill in the blanks in the following table and memorize the facts:

$$.... \text{ cu. in.} = 1 \text{ cu. ft.}$$
$$.... \text{ cu. ft.} = 1 \text{ cu. yd.}$$
$$.... \text{ cu. ft.} = 1 \text{ cord.}$$

The following facts are also important and should be memorized:

1. A ton of hard coal occupies about 35 cu. ft.
2. A ton of soft coal occupies about 42 cu. ft.
3. A ton of hay occupies nearly 500 cu. ft.
4. Four bu. of grain occupy nearly 5 cu. ft.
5. There are about $6\frac{1}{4}$ gal. in 1 cu. ft.
6. One cubic yard of gravel, or of sand, or of earth, is equal to a standard load.

EXERCISE 67

NOTE.—Use the cancellation method wherever possible.

1. A school-room is 32 ft. long, 25 ft. wide, and 12½ ft. high. How many cu. ft. of air does it contain?

2. If a coal bin is 14 ft. long, 10 ft. wide, and 6 ft. high, how many tons of hard coal will it hold? How many of soft coal?

3. Find the cost of digging a ditch 150 yd. long, 4 ft. wide, and 3 ft. deep @ 75c. a cu. yd.

4. Find the cost of digging a cellar 42 ft. long, 36 ft. wide, 9 ft. deep @ 25c. a cu. yd.

5. How many tons of hay are there in a mow which is 50 ft. long, 24 ft. wide, 15 ft. deep if the mow is full?

6. How many gal. will a rectangular tank 8 ft. deep, 4 ft. wide, and 4 ft. long hold?

7. How many bu. of wheat will a bin which is 5 ft. wide, 8 ft. long, and 7 ft. high hold?

8. Measure the length, the width, and the height of your school-room. Find how many cu. ft. of air it contains. How many cu. ft. of air are there for each pupil?

9. What is the value of a pile of wood 24 ft. long, 8 ft. wide, and 6 ft. high @ $14 a cord?

10. A bin 5 ft. wide, 5 ft. deep, and 7 ft. high is full of wheat. What is the value of the wheat @ $2·80 a bu.?

11. How many gal. of water are there in a tank which is 16 ft. long, 4 ft. wide, and 4 ft. deep if the tank is ¾ full of water?

12. A wagon box is 2 ft. 9 in. wide, and 10 ft. long. How high must the sides be to hold 1 cu. yd. of earth?

BOARD MEASURE

The unit used in the measurement of lumber is the **board foot,** by which is meant the *amount of lumber* contained in a rectangular piece of board 1 foot long, 1 foot wide, and 1 inch thick.

A board of other dimensions which contains the same amount of lumber as the board 1 ft. × 1 ft. × 1 in. also contains a board foot, for example, a board 4 ft. long, 3 inches wide, and 1 inch thick contains a board foot.

A board 12 ft. long, 12 in. wide, and 1 in. thick contains 12 board feet. A board 12 ft. long, 8 in. wide, and 1 in. thick contains 8 board feet, and a board 6 ft. long, 6 in. wide, and 1 in. thick contains 3 board feet.

If each of these boards were 2 in., $2\frac{1}{2}$ in., or $1\frac{1}{4}$ in. thick, it would contain 2, $2\frac{1}{2}$, or $1\frac{1}{4}$ times as many board feet.

Boards less than one inch thick are generally sold as inch lumber.

The dimensions of a board are given as follows:

$$11' \times 8'' \times 1\frac{1}{4}''$$

which means 11 ft. long, 8 in. wide, and $1\frac{1}{4}$ in. thick.

Lumber is bought and sold by the board foot, and the price is usually quoted at so much per **M.**, which means **per 1000 ft. board measure.**

How many cu. in. in a board foot?

Find the number of board feet in boards 1 in. thick whose other dimensions are:

1. $8' \times 4''$	6. $15' \times 12''$	11. $12' \times 6''$
2. $9' \times 3''$	7. $10' \times 6''$	12. $12' \times 5''$
3. $8' \times 3''$	8. $4' \times 3''$	13. $12' \times 7''$
4. $9' \times 4''$	9. $6' \times 4''$	14. $12' \times 11''$
5. $10' \times 12''$	10. $12' \times 9''$	15. $8' \times 6''$

Find the number of board feet in planks or scantlings whose dimensions are:

1. $12' \times 6'' \times 2''$	6. $12' \times 6'' \times 1\frac{1}{2}''$
2. $12' \times 3'' \times 4''$	7. $12' \times 10'' \times 2\frac{1}{3}''$
3. $12' \times 5'' \times 2''$	8. $12' \times 8'' \times 1\frac{1}{4}''$
4. $12' \times 2'' \times 2''$	9. $12' \times 6'' \times 1\frac{1}{4}''$
5. $12' \times 5'' \times 4''$	10. $12' \times 7'' \times 2\frac{1}{4}''$

EXERCISE 68

1. Find the number of board feet in boards, planks, or scantlings of the following dimensions:

(a) $15' \times 8'' \times 1\frac{1}{4}''$	(f) $9' \times 3\frac{1}{2}'' \times 4\frac{1}{2}''$
(b) $14' \times 10'' \times 1\frac{1}{2}''$	(g) $12' \times 5'' \times 2\frac{1}{4}''$
(c) $8' \times 6'' \times 4''$	(h) $16' \times 2\frac{1}{2}'' \times 2\frac{3}{4}''$
(d) $16' \times 15'' \times 4\frac{1}{2}''$	(i) $14' \times 4'' \times 4''$
(e) $10' \times 5'' \times 3\frac{1}{2}''$	(j) $15' \times 3\frac{1}{2}'' \times 2\frac{1}{2}''$

2. How much lumber is there in a stick 24 ft. long, 15 in. wide, and 12 in. thick?

3. A man bought 24 planks 15 feet long, 18 in. wide, and $3\frac{1}{2}$ in. thick. How much did they cost at 6c. a board foot?

4. What will be the cost of 30 pieces of pine scantling $12' \times 4'' \times 5''$ @ \$75 per M.?

5. What will be the cost of inch maple for a hardwood floor for a room 25 ft. by 20 ft. @ \$65 per M.?

6. Make out a bill for the following lumber @ \$50 per M.:

16 pieces	2 in. $\times$ 2 in. $\times$ 10 ft.	
15 "	4 in. $\times$ 5 in. $\times$ 12 ft.	
28 "	$2\frac{1}{2}$ in. $\times$ 8 in. $\times$ 14 ft.	
100 "	$1\frac{1}{2}$ in. $\times$ 6 in. $\times$ 10 ft.	
60 "	1 in. $\times$ 8 in. $\times$ 15 ft.	

PERCENTAGE

EXERCISE 69

Change each of the following fractions to an equivalent fraction with 100 as its denominator:

1. $\frac{1}{2}$	8. $\frac{1}{10}$	15. $\frac{21}{25}$
2. $\frac{1}{4}$	9. $\frac{3}{10}$	16. $\frac{13}{18}$
3. $\frac{3}{4}$	10. $\frac{7}{10}$	17. $\frac{41}{50}$
4. $\frac{1}{5}$	11. $\frac{13}{18}$	18. $\frac{69}{88}$
5. $\frac{2}{5}$	12. $\frac{3}{20}$	19. $\frac{75}{18}$
6. $\frac{3}{5}$	13. $\frac{17}{20}$	20. $\frac{1}{3}$
7. $\frac{4}{5}$	14. $\frac{9}{25}$	21. $\frac{2}{3}$

Simplify each of the following:

$$\tfrac{8}{100} \text{ of } \$243, \ \tfrac{3}{5} \text{ of } \$243, \ \tfrac{8}{40} \text{ of } \$243.$$

Which of these results is most easily obtained? Why?

The fractions with 100 as denominator are so convenient for calculating that business men use them a great deal. These fractions, known as **hundredths**, are given a special name—**per cent.**—and are denoted by a special symbol—**%**.

For instance, $\frac{5}{100}$ is called 5 per cent. and is written 5%; $\frac{25}{100}$ is called 25 per cent., and is written 25%; $\frac{12\frac{1}{2}}{100}$ is called 12½ per cent., and is written 12½%.

It is often necessary in business to obtain per cent. fractions which are equivalent to other fractions, for example,

$\frac{7}{10}$ changed to a per cent. fraction $\frac{70}{100} = 70\%$

$\frac{8}{25}$ "   "   "   " $\frac{32}{100} = 32\%$

$\frac{3}{4}$ "   "   "   " $\frac{75}{100} = 75\%$

$\frac{3}{8}$ "   "   "   " $\frac{37\frac{1}{2}}{100} = 37\frac{1}{2}\%.$

Again, it is often convenient to reduce some per cent. fractions, when making calculations with them, to their lowest terms, for example,

$$20\% = \tfrac{20}{100} = \tfrac{1}{5}; \quad 25\% = \tfrac{25}{100} = \tfrac{1}{4}; \quad 12\tfrac{1}{2}\% =$$
$$\frac{12\tfrac{1}{2}}{100} = \tfrac{25}{200} = \tfrac{1}{8}.$$

EXERCISE 70

1. Read the following:

   3%, 5%, 6%, 8%, 10%, 11%, 15%, 18%, 20%.

2. Write the per cents. in Question (1) as fractions, and reduce each fraction to its lowest terms.

3. Read the following:

   25%, 27%, 30%, 40%, 45%, 50%, 60%, 80%.

4. Write each of the per cents. in Question (3) as a fraction, and reduce each fraction to its lowest terms.

5. Read the following:

   $22\tfrac{1}{2}\%$, $33\tfrac{1}{3}\%$, $37\tfrac{1}{2}\%$, $62\tfrac{1}{2}\%$, $66\tfrac{2}{3}\%$, 75%, $87\tfrac{1}{2}\%$, 100%.

6. Write each of the per cents. in Question (5) as a fraction, and reduce each fraction to its lowest terms.

7. Reduce the following percentages to fractions in their lowest terms:

   5%, 10%, 20%, 25%, 30%, 40%, 50%, 60%, 70%, 75%, 80%, 90%, 100%.

8. Reduce the following percentages to fractions in their lowest terms:

   $33\tfrac{1}{3}\%$, $37\tfrac{1}{2}\%$, $62\tfrac{1}{2}\%$, $66\tfrac{2}{3}\%$, $87\tfrac{1}{2}\%$, 95%.

Example 1: Find $12\tfrac{1}{2}\%$ of \$1600.

$$12\tfrac{1}{2}\% = \frac{12\tfrac{1}{2}}{100} = \tfrac{25}{200} = \tfrac{1}{8}$$

$12\tfrac{1}{2}\%$ of \$1600 $= \tfrac{1}{8}$ of \$1600 = \$200.

Example 2: What per cent. of \$45 is \$1·50?

$$\tfrac{\$1.50}{\$45} = \tfrac{150}{4500} = \tfrac{1}{30}$$
$$\tfrac{1}{30} = \frac{3\tfrac{1}{3}}{100} = 3\tfrac{1}{3}\%.$$

ORAL EXERCISE

Find:

(a) 10% of $1200
(b) 20% of $4000
(c) 25% of $600
(d) 75% of 200 acres
(e) 100% of 450 tons

(f) $33\frac{1}{3}$% of 900 bu.
(g) $66\frac{2}{3}$% of 1200 automobiles
(h) $12\frac{1}{2}$% of 720 yd.
(i) $62\frac{1}{2}$% of $1600
(j) $11\frac{1}{9}$% of $2700

EXERCISE 71 (REVIEW)

1. Add and check each of the following Questions in 4 minutes:

(a) $167·34	(b) $436·74	(c) $829·94
43·26	234·62	4752·85
67·58	6758·41	661·54
987·53	98·75	8675·83
49·76	497·61	672·44
8·39	839·84	29·73
657·61	15·28	456·19
748·16	755·18	5841·28
62·39	2639·43	75·41
822·16	288·65	746·39
721·50	7215·94	3824·67
18·52	3589·28	127·05
15·17	81·25	436·23
274·38	4728·93	8830·47
135·86	13·49	921·11
333·33	6·78	5342·39

2. Subtract and test two questions in a minute:

(a) 9500832	(b) $28050·72	(c) 3684021
5362834	8493·84	986518

(d) 970005	(e) $79002·31	(f) $27054·20
609099	57220·90	6474·23

3. Multiply and test one question in two minutes:

    (a) 8984   (b) 7564   (c) 8056
        2169   6034   6907

    (d) 6528   (e) 8463   (f) 5829
        4539   2765   4783

4. Find the quotient and test each of the following questions in two minutes:

    (a) 534)9268638   (b) 890)62752120
    (c) 489)$443388·81   (d) 989)$540401·80

5. Find the value of:

(a) $(36 \times 217 \times 528 \times 75) \div (84 \times 396 \times 155 \times 27)$
(b) $(49 \times 75 \times 156 \times 95) \div (108 \times 105 \times 171 \times 39)$

6. Find the prime factors of:

    (a) 4235   (b) 2142   (c) 5145   (d) 1331

7. Find the G.C.M. of (a) 408 and 544
                  (b) 396 and 693

8. Find the L.C.M. of (a) 32, 52, 65
               (b) 38, 95, 133

9. Reduce the following fractions to their lowest terms:

    (a) $\frac{165}{231}$ (b) $\frac{168}{120}$ (c) $\frac{720}{840}$ (d) $\frac{885}{885}$

10. Simplify:

    (a) $\frac{1}{3} + \frac{3}{4} - \frac{5}{6}$
    (b) $\frac{2}{5} + \frac{5}{6} + \frac{7}{12}$
    (c) $\frac{7}{8} - \frac{2}{3} + \frac{3}{4}$
    (d) $18\frac{7}{8} + 9\frac{1}{6}$
    (e) $29\frac{1}{4} - 14\frac{2}{5}$
    (f) $\frac{14}{22} \times \frac{66}{84} \times \frac{45}{77}$
    (g) $\frac{4}{7}$ of $(\frac{3}{8} + \frac{5}{7})$
    (h) $\frac{7}{15} \times \frac{4}{21} \div \frac{25}{36}$
    (i) $\frac{49}{26} \times \frac{75}{42} \times \frac{108}{26} \times \frac{78}{18}$
    (j) $\dfrac{7\frac{7}{13} \times 5\frac{2}{3}}{8\frac{1}{6} \times 9\frac{5}{7}}$

11. Find the area in sq. ft. of each of the following rectangles:

> (a) 6 ft. 4 in. by 3 ft.
> (b) 9 yd. 2 ft. by 3 ft. 6 in.
> (c) 152 in. × 11 ft.

12. Find the length in feet of each of the following rectangles:

> (a) Area 720 sq. ft., width 30 ft.
> (b) " 365 "   " 18¼ ft.
> (c) " 372 "   " 8 yd.

13. Find the vol. in cu. ft. of each of the following:

> (a) A rectangular block of stone 18 ft. × 7 ft. × 4½ ft.
> (b) A pile of wood 25 ft. long, 8 ft. wide, 7 ft. high. Find also the number of cords.
> (c) The amount of gravel in a pit 27 ft. 6 in. long, 11 ft. 3 in. wide, and 12½ ft. deep.

14. Find the number of feet, board measure, in the following:

> (a) 8 planks 2 in. thick, 10 ft. long, 12 in. wide
> (b) 2 sticks of timber 18 ft. long, 10 in. wide, 9 in. thick
> (c) 8 beams 10 in. × 12 in. and 14 ft. long.

15. Find the number of gallons which rectangular tanks of the following dimensions will hold:

> (a) 6 ft. × 2¼ ft. × 1 ft. 8 in.
> (b) 12½ ft. × 15½ ft. × 6½ ft.

16. Find the number of tons of hard coal which bins of the following dimensions will hold:

> (a) 13 ft. × 7 ft. × 5 ft.
> (b) 10 ft. 6 in. × 6 ft. 6 in. × 6 ft.

17. Find the number of tons of hay which mows of the following dimensions will hold:

> (a) 50 ft. long, 18 ft. wide, 14 ft. deep
> (b) 36 ft. long × 20 ft. wide, 15 ft. deep.

18. Find the number of bu. which bins of the following dimensions will hold:

> (a) 12 ft. × 6 ft. × 5 ft.
> (b) 15½ ft. × 8½ ft. × 5 ft.
> (c) 16¾ ft. × 9½ ft. × 5½ ft.

19. Find the number of tons of soft coal which a bin 9 ft. × 5¼ ft. × 6½ ft. will hold.

20. Find the number of cords of wood in piles of the following dimensions:

> (a) 24 ft. × 12 ft. × 6 ft.
> (b) 48 ft. × 16 ft. × 5 ft.
> (c) 32 ft. × 20 ft. × 7 ft.

21. To what fraction in its lowest terms is each of the following percentages equal?

> (a) 6⅓%, (b) 37½%, (c) 25%, (d) 33⅓%, (e) 62½%,
> (f) 83⅓%, (g) 87½%, (h) 97½%?

22. To what per cent. is each of the following fractions equal?

> (a) ¾, (b) ⅞, (c) ⅜, (d) ⅝, (e) 9/7, (f) ⅚, (g) 7/12, (h) ⅔, (i) ½
> (j) ⅗?

23. What is:

> (a) 12½% of $720?   (d) 62½% of $968?
> (b) 14²⁄₇% of $840?   (e) 87½% of $648?
> (c) 33⅓% of $573?

24. A binder costs an agent $175. If he sells it to gain 30% of his cost, find the selling price.

25. A real estate agent sold a house for $3750 and received 2% of the selling price for his services. How much did the owner of the house receive?

26. A grocer marks his goods so as to receive a gain of 15% on the selling price. If the grocer's sales for a week are $356·75, find his gain for the week.

27. E. Murray bought of Sherwin and Ball, March 1st, 1919:

> 8 lb. Rio Coffee @ 45c.
> 4 lb. tea @ 65c.
> $8\frac{1}{2}$ doz. eggs @ 72c.
> $5\frac{3}{4}$ lb. butter @ 68c.
> $\frac{1}{2}$ gal. maple syrup @ $2·40
> $4\frac{3}{4}$ lb. cheese @ 36c.
> $\frac{3}{4}$ doz. cans tomatoes @ $2·60 a doz.

Make out Murray's bill and receipt it.

28. Harry decided on Sept. 1st. to keep an account of his earnings and expenses. His mother gave him $5·00 in cash. He worked at a store on Saturdays, Sept. 5, 12, 19, and 26, and earned $1·00 each day. After school hours, on Sept. 7, 10, 14, and 25 he ran errands and earned 25c. each day. On Sept. 3rd, he paid for a book, 50c. On Sept. 8th, he paid $1·00 for a ball, on Sept. 11th, 75c. for a tie. On Sept. 21st, $1·25 for a cap.

Make out his account and find his balance on Oct. 1st.

29. You have had the following transactions with Murray Keith and Co. Make out your account as Murray Keith and Co. would:

Debits: April 4th, groceries $15·90; April 5th, lumber, $190·72; April 11th, dry-goods, $23·15; April 12th, wagon, $90·00; April 27th, sugar, $20·70; a buggy, $129·00; May 7th, groceries, $40·05; May 12th, coat, $100; May 20th, lumber, $189·00.

Credits: April 10th, labour, $129·71; April 14th, cash, $25·00; April 21st, labour, $29·70; April 28th, cash, $147; May 7th, labour, $270·20; May 19th, stone work, $175·39; May 28th, cash, $70·00.

30. Find the total cost of 1 cu. yd. of cement wall.

> $1\frac{1}{3}$ cu. yd. of gravel @ $2 per cu. yd.
> 2 bbl. of cement @ $2·80 a bbl.
> Labour, $2·00.

31. Find the cost of a cement wall which is 44 ft. long, 8 ft. high, and 10 inches thick at $9·85 a cu. yd.

32. Find the cost of the footing for a cement wall, if the footing is 18 in. wide, 6 in. high, and 46 ft. long @ $9·85 a cu. yd.

33. Find the number of loads of earth removed in excavating for a cellar 26 ft. wide, 39 ft. long, and 5 ft. deep, if each load contains $1\frac{1}{2}$ cu. yd.

34. Estimate the number of cords of wood in 3 piles each 18 in. wide, 60 ft. long, and 6 ft. high. If 4-foot wood is worth $15·00 a cord, what should the 18-in. wood be worth per cord?

35. How many cubical boxes 3 ft. 6 in. to a side can be placed in a store-room 14 ft. × 14 ft. × 14 ft.?

36. At $1·25 a sq. yd. find the cost of a tin roof each side of which is 36 ft. by 22 ft. 6 in.

37. What will it cost to put a fence around the outside of a farm which contains 120 acres and is 60 rods wide, at $2·75 a rod?

38. Find the cost at $45 per M. of:

    (a) 50 boards 16 ft. long, 12 in. wide, $1\frac{1}{2}$ in. thick

    (b) 75 boards 18 ft. long, 10 in. wide, $1\frac{3}{4}$ in. thick

    (c) 120 studding 2″ × 4″ × 11′

    (d) 125 sleepers 3″ × 10″ × 28′

    (e) 325 planks 3″ × 14″ × 16′

    (f) 70 sills 10″ × 12″ × 30′.

SECTION IV

DECIMALS

Give the place name for each digit in the number 44444. Compare the value of the 4 representing units with the 4 immediately to the left of it, with the 4 two places to the left of it, with the 4 three places to the left of it.

If a digit, 4, be put to the right of the 4 representing units, its value should be what fraction of the value of the 4 units?

If a digit, 4, be placed two places to the right of the 4 representing units, its value will be what fraction of the value of the 4 units?

Hence, digits to the right of the units' digit represent fractions. What is the denomination of the fraction represented by the 4 immediately to the right of the units' digit? Of the fraction represented by the digit two places to the right of the units' digit? Three places to the right?

Ordinary notation is, therefore, extended to include fractions whose denominators are 10 or some *power* of 10, that is 10, 100, 1000, 10000, &c., &c.

Such fractions are called **decimal fractions** or simply **decimals.**

All other fractions are called **common** or **vulgar fractions.**

In order to show whether a digit represents an integer or a fraction, a dot (·) called the **decimal point** is placed to the right of the units' digit. All figures to the left of this point will represent integers, and all to the right, fractions.

Thus in 36·45 the 3 is tens, the 6 is units, the 4 is tenths, and the 5 is hundredths. The value of each digit depends, as in whole numbers, upon the place it occupies.

NOTATION AND NUMERATION OF DECIMALS

To read and write decimals the Table for ordinary numeration and notation is extended as follows:

Hundred-Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
			5	6	7	3	1	2	4		

In the number ·3124, what is the place value of the 3? of the 1? of the 2? of the 4?

Expressing the value of each of these digits in the form of a fraction the results are:

$$\tfrac{3}{10}, \ \tfrac{1}{100}, \ \tfrac{2}{1000}, \ \text{and} \ \tfrac{4}{10000}.$$

These fractions, when brought to the same denominator, become $\tfrac{3000}{10000}, \ \tfrac{100}{10000}, \ \tfrac{20}{10000}, \ \tfrac{4}{10000}.$

And when summed, the result is $\tfrac{3124}{10000}$.

Hence, the number ·3124 is read as:

Three-thousand one hundred and twenty-four ten thousandths.

The number 567·069 is read as:

Five hundred and sixty-seven *and* sixty-nine thousandths.

A number which is a decimal, therefore, is read by giving it the place name of its right-hand digit.

Note that the reading of the whole number and the reading of the fraction is connected by the word *and*.

ORAL EXERCISE

1. Read the following: 3, ·7, 1·3, 24·6, 306·8, 2001·4.

2. Read the following: ·42, ·75, ·08, 3·51, 2·04, 18·63, 27·01, 30·87, 60·03, 218·42, 500·05, 30001·08.

3. Read the following: ·825, ·416, ·307, ·006, 2·412, 3·005, 64·157, 309·043, 700·009, 6020·075.

4. Read the following: ·4165, ·0216, ·0035, ·0008, 756·3168, 4218·0032.

5. Write: Six-tenths, five *and* three tenths, seventy *and* two tenths, six hundred *and* three tenths, eighty hundredths, seven *and* fifty-six hundredths, nine hundredths, three hundred and four *and* five hundredths, seven thousandths, eleven *and* eight hundred and fifty-seven thousandths, six thousand two hundred *and* eight ten-thousandths, five thousand two hundred and nine *and* twenty-seven ten-thousandths.

6. Express as tenths: 5 units, 3 units, 4 units and 3 tenths.

7. Express as hundredths: 9 units, 7 units, 3 tenths, 8 tenths, 5 units and 7 tenths, 3 units 6 tenths 4 hundredths.

8. Express as thousandths: 4, ·03, 5, 3·2, 92·01, 12.

9. Write in decimal form:

$\frac{7}{100}$, $\frac{15}{1000}$, $\frac{4}{1000}$, $\frac{32}{10}$, $\frac{805}{10000}$, $\frac{327}{100000}$, $16\frac{7}{100}$, $365\frac{8}{1000}$.

10. Examine: 6·3, 6·30, 60·3, and 6·03.

State what change is made in the value of a number when a cipher is placed before the integral part; after the decimal part; between the decimal point and the first significant digit to the left; between the decimal point and the first significant digit to the right.

NOTE.—The digits 1, 2, 3, 4.....9 are called *significant* digits.

ADDITION AND SUBTRACTION OF DECIMALS

In the addition of integers how are the addends placed with respect to one another? In subtraction how is the subtrahend placed with respect to the minuend?

The same placing is observed in the case of addition and subtraction of decimals. Units are placed under units, tens under tens, tenths under tenths, hundredths under hundredths, &c. This arrangement will be carried out most easily by placing the decimal points in a perpendicular row.

Example 1: Find the sum of 6·32, 45·817, ·09, 867·8256, 3·14.

These numbers are placed for addition as follows:

$$
\begin{aligned}
6&\cdot32 \\
45&\cdot817 \\
\cdot&09 \\
867&\cdot8256 \\
3&\cdot14 \\
\hline
923&\cdot1926
\end{aligned}
$$

The digits in the same columns are added, 10 units of any order being equal to 1 unit of the next higher order.

Example 2: From 365·04 take 98·736.

The work is set down thus:

$$
\begin{aligned}
365&\cdot04 \\
98&\cdot736 \\
\hline
266&\cdot304
\end{aligned}
$$

From what has the 6 thousandths been taken?

EXERCISE 72

Find the value of:

1. ·6 + ·9 + ·46 + ·38 + ·834 + ·416.

2. 9·3 + 18·7 + 625·4 + 3·8 + 53·8.

3. 29·84 + 15·83 + 26·37 + 69·45 + 37·98.

4. 34·46 + 87·5 + 9·432 + 78·6 + 6·7.

5. 439·6 + 5376·84 + 6397·882 + 3261·984.

6. $3·90 + $87 + $14·04 + $·015 + $26 + $9·875.

7. $28·5 + $37·876 + $715 + $0·46 + $6·218 + $0·75.

8. 49·4 − 3·86217,   1003 − 5·846,   10 − ·00423, 937·436 − 488·679.

9. 53·8 + 8·64 − 46·008 − 3·4 + 19 − ·0547.

10. $64·20 + $38·124 − $0·85 − $23·1 + $5 − $·0575.

11. From Montreal to Ottawa is 115·7 miles, from Ottawa to North Bay 244·3 miles, from North Bay to Sudbury 79·2 miles, and from Sudbury to Fort William is 556·3 miles. How far is it from Montreal to Fort William?

12. In the first of three farms there are 87·6 ac. of land, in the second there are 14·9 ac. more than in the first, and in the third there are 42·57 ac. less than in the first and second together. How many acres of land are there in the three farms?

13. A steamship goes 385·4 miles the first day, 296·7 mi. the second day, 347 mi. the third day, 398·875 mi. the fourth day, and 286·35 mi. the fifth day. How far has it yet to go to complete a voyage of 2000 mi.?

14. Out of 100 parts of bread, 8·9 parts are nitrogenous matter, 56·7 parts starchy, 4·1 parts fatty, and 1·1 parts salty. The rest is water. How many parts of water are there?

15. Of apple pie ·031 is nitrogenous, ·428 starchy, ·018 salty, and ·425 is water. The rest is fatty. How much of apple pie is fatty?

MULTIPLICATION OF DECIMALS

$\cdot 6 \times 7 = \frac{6}{10} \times 7 = \frac{42}{10} = 4\frac{2}{10} = 4\cdot 2.$

$\cdot 6 \times \cdot 7 = \frac{6}{10} \times \frac{7}{10} = \frac{42}{100} = \cdot 42.$

$6 \times 7 = 42.$

$\cdot 37 \times \cdot 5 = \frac{37}{100} \times \frac{5}{10} = \frac{185}{1000} = \cdot 185.$

$37 \times 5 = 185.$

$\cdot 64 \times \cdot 36 = \frac{64}{100} \times \frac{36}{100} = \frac{2304}{10000} = \cdot 2304.$

$64 \times 36 = 2304.$

$3\cdot 52 \times 1\cdot 43 = \frac{352}{100} \times \frac{143}{100} = \frac{50336}{10000} = 5\cdot 0336.$

$352 \times 143 = 50336.$

It is evident from these examples that the only difference between the multiplication of whole numbers and the multiplication of decimals is that in the latter the decimal point has to be placed.

Compare the number of digits in the decimal part of each of the above products with the number of digits in the decimal parts of the multiplicand and the multiplier.

Example 1: Multiply $37\cdot 85$ by $4\cdot 7$.

```
  37·85
   4·7        What is the number of decimal places in
 _____       the multiplicand and the multiplier?
  26·495
 151·40       How many decimal places should there be
 _____       in the product?
 177·895
```

Explanation: In the multiplicand $37\cdot 85$ the 5 is hundredths, $\frac{5}{100}$, and the 7 of the multiplier is tenths, $\frac{7}{10}$, $\frac{5}{100} \times \frac{7}{10} = \frac{35}{1000} = \cdot 035.$

$\therefore$ The 5 of the product is thousandths, the 9 is hundredths, the 8 is tenths, and the first 7 is units.

$\therefore$ The decimal point is placed between the 7 and the 8.

EXERCISE 73

1.   Multiply:

(a) 3·7 by 8   (i) ·348 by ·05

(b) 4·59 by 9   (j) 476·54 by 3·62

(c) 78·643 by 67   (k) 354 by ·726

(d) 5816 by ·45   (l) 4375 by 5·77

(e) 8·3 by 7·6   (m) ·819 by 6·14

(f) 9·42 by 5·1   (n) 12·28 by 3·9

(g) 92·77 by 34·8   (o) 2·43 by ·008

(h) ·273 by 9·5   (p) ·0842 by 1·364

2. Multiply 4·32156 by 10, by 100, by 1000. What change does each of these multipliers make in the position of the decimal point? How, then, can the product be written down at once for a decimal multiplied by 10? by 100? by 1000? by 10000?

3. Write the product of 5·4637 by 1000, 137·846 by 100, 29·7836 by 10000.

4. The length of a metre is 39·37 in. How many inches are there in 7·5 metres?

5. How many oz. are there in ·4375 lb.?

6. How many feet are there in ·525 miles?

7. How many pounds are there in ·348 tons?

8. Reduce ·625 bu. to gal.

9. Reduce ·34 wk. to hr.

10. A merchant sells goods at a gain of ·125 of their cost. What will he receive for goods which cost him $480?

11. One pound of milk makes ·1087 lb. of cheese. How many lb. of cheese can be made from 360 lb. of milk?

12. What will be the cost of 80·4 tons of coal at $11·50 a ton?

13. A teacher paid ·35 of his salary for board, ·18 for clothing, ·07 for travel, ·1 for incidentals, and had $300 left. What was his salary?

14. If the value of the English pound sterling is $4·8665, what is the value in our money of £27?

15. The normal value of the English pound in Canadian money is $4·8665. If after-war conditions have decreased the value of the pound to $3·975, what difference would this loss make in the value of £75?

16. If the two sides of a rectangular field are, respectively, 96·8 ft. and 137·6 ft., what is the perimeter of the field?

EXERCISE 74 (REVIEW)

1. During 8 hr. a freight train made the following mileages: 32·15, 28·375, 15·687, 20·2, 15·63, 17·5, 8·95, and 21·3. How far did the train run during the 8 hr.?

2. A coal dealer received 9 carloads of coal of the following tonnages: 24·6, 28·785, 29·6375, 24·95, 25·125, 27·137, 31·038, 30·456, 28·555.

What was the total tonnage received? What was the average tonnage in each car?

3. 54 cu. in. of soil in its natural state weighed 1·94 lb. After being thoroughly dried, it weighed 1·459 lb. How much moisture passed off in drying?

4. 1·135 lb. of dry beans soaked for 24 hr. weighed 2·212 lb. What was the weight of water taken up by the beans?

5. Find the cost of 8475 cu. ft. of gas at $1·50 per thousand cu. ft.

6. Find the value of 7·642 tons of coal @ $13·50 a ton.

7. Find the value of a load of hay weighing 2·543 tons @ $22·50 a ton.

8. Find the value of ·689M board feet of lumber @ $45·75 per M.

9. Find the cost of roofing a barn containing 25·47 hundred sq. ft. @ $4·75 per 100 square feet.

PERCENTAGE

ORAL EXERCISE

1. Express the following fractions as per cents.:
   $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{7}{20}$, $\frac{11}{20}$.

2. Express the following fractions as per cents.:
   $\frac{1}{25}$, $\frac{11}{25}$, $\frac{87}{50}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$.

3. Express the following per cents. as fractions in their lowest terms:
   5%, $12\frac{1}{2}$%, 10%, $14\frac{2}{7}$%, $9\frac{1}{11}$%, 20%.

4. Express the following per cents. as fractions in their lowest terms:
   25%, $33\frac{1}{3}$%, $37\frac{1}{2}$%, $62\frac{1}{2}$%, $66\frac{2}{3}$%, $83\frac{1}{3}$%, $87\frac{1}{2}$%.

Find the value of:

5. 20% of 800, 7% of 900, 12% of 1200

6. $8\frac{1}{3}$% of 480, $11\frac{1}{8}$% of 729, $12\frac{1}{2}$% of 648

7. $33\frac{1}{3}$% of 396, $87\frac{1}{2}$% of 568, $62\frac{1}{2}$% of 728

8. $16\frac{2}{3}$% of 366, 75% of 968, 2% of 750.

Men in business, at different times in the year, advertise "Bargain Sales." To induce people to buy, these business men reduce the regular selling price of their goods by a certain per cent. This reduction is called **a discount,** and goods are said to be sold at *a certain per cent. discount.* For example, suppose the regular selling price of an article is $25·00 and the rate of discount is 20% = $\frac{1}{5}$ — the reduced selling price is $\frac{4}{5}$ of $25 = $20.

EXERCISE 75

1. Find the reduced selling price from the following sales as advertised:

Regular price			Per cent. of discount	Reduced selling price
(a) Overcoats	worth	$75	$33\frac{1}{3}\%$	
(b) Men's suits	"	$84	$16\frac{2}{3}\%$	
(c) Boys' suits	"	$36	$12\frac{1}{2}\%$	
(d) Boys' overcoats	"	$27	$33\frac{1}{3}\%$	
(e) Hats	"	$8·40	$12\frac{1}{2}\%$	
(f) Caps	"	$2·50	25%	
(g) Socks	"	$1·75	20%	
(h) Ties	"	$2·25	10%	

2. Find the gain or loss and the selling price:

    (a) Cost is $75, gain is 20%.

    (b) Cost is $96, loss is $33\frac{1}{3}\%$.

    (c) Cost is $225, gain is $12\frac{1}{2}\%$.

    (d) Cost is $132·50, gain is 10%.

TO FIND WHAT PER CENT. ONE NUMBER IS OF ANOTHER NUMBER

Example 1: What per cent. of $3\frac{1}{2}$ yd. is $\frac{1}{2}$ yd.?

    $\frac{1}{2}$ yd. is the fraction $\frac{\frac{1}{2}}{3\frac{1}{2}}$ of $3\frac{1}{2}$ yd.

    $\frac{\frac{1}{2}}{3\frac{1}{2}} = \frac{1}{2} \times \frac{2}{7} = \frac{1}{7} = 14\frac{2}{7}\%$.

    $\therefore \frac{1}{2}$ yd. is $14\frac{2}{7}\%$ of $3\frac{1}{2}$ yd.

Example 2: What per cent. of 1 bu. 2 pk. is 3 qt.?

    1 bu. 2 pk. = 6 pk. = 12 gal. = 48 qt.

    3 qt. is $\frac{3}{48}$ of 48 qt.

    $\frac{3}{48} = \frac{1}{16} = 6\frac{1}{4}\%$,

    $\therefore$ 3 qt. is $6\frac{1}{4}\%$ of 1 bu. 2 pk.

EXERCISE 76

1. What per cent. of:

(a) 25 is 15?   (g) 4 is $\frac{3}{5}$?

(b) 32 is 20?   (h) $2\frac{1}{2}$ is $1\frac{3}{4}$?

(c) 65 is 52?   (i) 11 gal. is 4 qt.?

(d) 96 is 60?   (j) $3\cdot20 is 64c.?

(e) 25 is $6\frac{1}{4}$?   (k) $8\cdot40 is $1\cdot40?

(f) $\frac{7}{5}$ is $\frac{5}{5}$?   (l) $42\cdot90 is $7\cdot15?

2. Find the per cent. discount offered in selling:

    (a) Ladies' coats   price $27·75 for $18·50

    (b) Girls'   "   " $20·50 " $16·40

    (c) Ladies' shoes   " $16·50 " $11·00

    (d) Girls'   "   " $ 9·75 " $ 7·80

    (e) Ladies' hats   " $26·50 " $21·20

    (f) Girls'   "   " $ 9·60 " $ 8·40.

3. A grocer marks his goods so that he can reckon his profit as $12\frac{1}{2}\%$ of his sales. What is his profit for each of the three days when his sales were $39·75, $47·55, $62·78?

4. A man owns a farm valued at $9600. His annual taxes are $86·40. How much must he make each year to clear 8% on the value of his property after paying taxes?

5. A boy's salary was increased from $45 to $55 a month. Find the per cent. of the increase in his salary?

6. One cow in a dairy gives 28 lb. of milk daily, which tests $3\cdot1\%$ of butter fat. Another cow gives daily 24 lb. of milk, which tests $3\cdot7\%$ of butter fat. If butter fat is worth 75c. a lb., what is the difference in the returns from these two cows in one week?

TO FIND A NUMBER WHEN A CERTAIN PER CENT. OF IT IS GIVEN

ORAL EXERCISE

1. Find the total if:
   - (a) $12\frac{1}{2}\%$ of it is $4·80
   - (b) $14\frac{2}{7}\%$ of it is $1·32
   - (c) $37\frac{1}{2}\%$ of it is $39
   - (d) $33\frac{1}{3}\%$ of it is $42
   - (e) 12% of it is $84
   - (f) $62\frac{1}{2}\%$ of it is $75.

2. If $150 is $8\frac{1}{3}\%$ of A's salary, find A's salary.

3. After a battle, 71% of a regiment, or 639 men, were left. How many men were in the regiment before the battle?

4. I withdrew from my savings deposit $175, which is $83\frac{1}{3}\%$ of my deposit. What was my balance before the withdrawal?

5. A teacher's expenses are $39·50 a month. This is $16\frac{2}{3}\%$ of his monthly salary. What is the amount of his monthly savings?

6. The number of pupils in attendance at school is 768, which is 96% of the enrolment. What is the enrolment?

EXERCISE 77 (REVIEW)

1. What is the percentage increase in price in each of the following:
   - (a) Milk goes up from 5c. a pint to 8c. a pint.
   - (b) Butter goes up from 45c. a lb. to 75c. a lb.
   - (c) Bread goes up from 7c. a loaf to 12c. a loaf.
   - (d) Sugar goes up from 5c. a lb. to 14c. a lb.
   - (e) Ham goes up from 30c. a lb. to 75c. a lb.
   - (f) Cheese goes up from 18c. a lb. to 38c. a lb.
   - (g) Potatoes go up from $1·75 a bag to $2·75 a bag.
   - (h) Apples go up from $3 a bbl. to $7 a bbl.

2. By paying cash Mrs. Hay gets cheese for 25c. a lb. instead of 28c. a lb. What per cent. of the 28c. does she save by paying cash?

3. The material for a side-walk costs $240, the labour costs $\frac{2}{3}$ as much. If the work completed costs $600, what per cent. of the cost of material and labour is the profit?

4. A merchant has invested in his building and store $9000. After paying all expenses, he finds he has cleared $12\frac{1}{2}\%$ on the cost of the building and store. What are his profits?

5. How much butter fat is there in 375 lb. of milk brought to a creamery, the creamery test showing that $3\cdot9\%$ of the weight of the milk is butter fat?

6. A farmer takes 340 lb. of milk to a creamery and is paid 75c. a pound for the butter fat. The tests show that the butter fat is $3\cdot8\%$ of the weight of the milk. How much does the farmer receive?

BANKS AND BANKING

Tell what you know of the Penny Savings Bank.

Give the names of Canadian Banks which you know.

Of what service to the public are banks?

A **bank** is a business institution for the receiving and safe-keeping of money and for the making of loans. It also cashes cheques, collects accounts, exchanges money of our country for the money of other countries, and issues notes (bank bills) which circulate as money.

Examine a five-dollar or ten-dollar bill. Who issued the note?

The Banks of Canada are corporations chartered by law and are subject to supervision by the Government of Canada, as a means of protecting the interests of depositors.

A person may open two kinds of accounts with a bank: the **Savings Account,** on which interest is allowed on the minimum monthly balances, and the **Current Account,** on which no interest is allowed.

NOTE.—Interest is money paid for the use of money.

OPENING AN ACCOUNT WITH A BANK

Suppose you have saved from your earnings 25 dollars, made up as follows:

2 five-dollar bills; 4 two-dollar bills; 5 one-dollar bills; two dollars in silver.

Deposit Slip—Savings Account			
SCHOLASTIC BANK			
Credit—J. R. Hill			
Deposited by—J. R. Hill			
2	of	Feb.	1920
		5 × 1	5
		4 × 2	8
		4	
		2 × 5	10
		10	
		20	
		50	
		100	
S.M.		500	
			23
	Coin		2
	Cheques		
			25 00

You wish to deposit this money in a bank for safe keeping and for the interest allowed.

In order to open with a bank either a Savings Account or a Current Account, you first fill out a **deposit slip,** as is here shown, and give it, with the 25 dollars, to the Receiving Teller. He counts the money and examines the slip to see that it and the money correspond. If they do, he initials the slip and passes it on to the Ledger Keeper, who opens an account

for you in the Bank's Ledger Book. You are then given a small Pass Book which should always be presented to the Teller or Ledger-keeper when a deposit or a withdrawal is made, in order that the dates and amounts of all deposits and withdrawals may be entered.

Examine this book whenever you deposit or withdraw money. This will prevent mistakes being made.

When you open an account with a bank, you are asked to write your name and address in a book kept by the bank, so that the bank may have your signature for identification.

EXERCISE 78

Make out deposit slips for the following deposits, in any bank you wish:

1. Jan. 3rd, 1920

    $7 \times \$5$, $4 \times \$2$, $9 \times \$1$, silver $\$6 \cdot 45$

2. Jan. 17th, 1920

    $2 \times \$10$, $6 \times \$5$, $7 \times \$2$, $23 \times \$1$, silver $\$15 \cdot 75$

3. Jan. 31st, 1920

    $1 \times \$20$, $4 \times \$10$, $9 \times \$5$, $35 \times \$1$, silver $\$8 \cdot 90$

4. Feb. 14th, 1920

    $2 \times \$50$, $4 \times \$20$, $15 \times \$2$, $13 \times \$1$, silver $\$19 \cdot 35$

5. Feb. 28th, 1920

    $1 \times \$100$, $3 \times \$50$, $5 \times \$20$
    Cheques, $\$74 \cdot 48$, $\$15 \cdot 62$, $\$39 \cdot 75$

6. Mar. 13th, 1920

    $2 \times \$500$, $7 \times \$2$, $9 \times \$1$
    Cheques $\$5867 \cdot 32$, $\$289 \cdot 75$

THE BANK CHEQUE

You have learned how to deposit money to your credit in a bank. You also need to know how to withdraw money from your bank account. This is done by means of a **cheque,** which is a written order to the bank signed by yourself to pay you the sum which you desire. Cheque forms are usually made up into small books, of which the following is a sample page:

STUB	CHEQUE
No. 29	*No.* 29 Stratford, Dec. 5, 1919
Date Dec. 5th, 1919	THE DOMINION BANK
Favour of	*Pay* Myself *or Bearer*
Myself	Fifteen . . . . $\frac{75}{100}$ *Dollars*
For Expenses	15\frac{75}{100}$
Amount 15\frac{75}{100}$	R. J. Fuller

Detach the right-hand portion of the cheque and present it to the teller of the bank, who will give you cash to the amount of $15\frac{75}{100}$ dollars. The left-hand part, called the "stub," is not detached from the cheque book, but is kept by you for reference.

You may use a cheque for another purpose besides withdrawing money for yourself. If you owe money to another person, you may pay your debt by issuing a cheque in his favour against the balance to your credit in the bank. The form of such a cheque is as follows:

11

No. 30	No. 30
Date Dec. 6th, 1919	Stratford, Dec. 6th, 1919
Favour of	THE DOMINION BANK
F. M. Allen	Pay F. M. Allen or Order
For Book-case	Thirteen............$\frac{50}{100}$ Dollars
Amount $13·50	$13·50 R. J. Fuller

The detached part is given to F. M. Allen, the person whom you owe. He presents it at the bank and receives the cash.

The above cheque, No. 30, differs from No. 29 in that the name of F. M. Allen is substituted for "myself" and the word "Order" replaces the word "Bearer." Why?

Cheques which you issue against the balance of your current account are returned to you by the bank. They serve as receipts for debts paid.

Cheques issued against a Savings Account are not returned. Hence receipts for payments should be asked for.

EXERCISE 79

Write the cheques, with stubs attached, for the following transactions. Select your own dating and name of bank for the cheques.

1. You draw $2·75 from your bank account for spending money.

2. You purchase a suit of clothes from R. S. McIntyre, Clothier, and give him a cheque for $37·50.

3. For repairs to his automobile, Wm. Fuller gives Arthur Hale a cheque for $17·85.

4. J. W. Chalmers buys a binder from S. B. Irwin for $225·00. J. W. Chalmers pays for it by cheque.

5. Richard Fuller sends a cheque for $27·89 to John Hill to pay for groceries purchased.

6. F. W. Mercer sells Douglas Fraser an automobile for $1200. Fraser gives a cheque in payment.

7. You purchase a pair of boots for $7·50 from J. W. Robb. You give in payment your cheque.

8. You draw $17·50 from your bank account to meet ordinary expenses.

THE PASS BOOK

All deposits and cheques issued against your balance are entered in your **pass book**, the form of the first page of which appears below:

THE SCHOLASTIC BANK
Savings Bank Department

Account No. 752　　*at*　　Toronto Branch

Date 1919	Particulars	Initials	Dr.		Cr.		Balance	
Sept. 1		G			315	38	315	38
" 3		G	12	08			303	30
			8	60			294	70
" 8		G	225	00			69	70
" 15		G			173	00	242	70
" 16		G	75	00			167	70
" 19		G	12	48			155	22
" 26		G	72	80			82	42
" 30		G	9	70			72	72

NOTE:

1. You are credited with all deposits. Hence, all deposits are entered in the *Cr.* column.

2. You are debited with all withdrawals. Therefore, all cheques which you issue appear in the *Dr.* column.

3. The differences between the amounts in the Dr. and Cr. columns go in the column for *balances*. There should be a balance for every debit or credit entry.

Examine the balances. Select the smallest one. This one is called the *minimum monthly balance* for the month of September. It is on this balance that the chartered banks will allow interest at 3% per annum for that month.

<div align="center">EXERCISE 80</div>

Find the minimum monthly balances:

1. Balance May 1st. $619·38

May	8th.	withdrew	$ 25·00
"	9th.	"	500·00
"	11th.	deposited	183·00
"	13th.	withdrew	3·00
"	22nd.	"	12·50
"	23rd.	"	75·00
"	26th.	"	100·00
"	27th.	"	25·00

2. Balance Sept. 1st. $315·38

Sept.	3rd.	withdrew	$ 12·08
"	4th.	"	8·60
"	7th.	"	225·00
"	14th.	deposited	173·00
"	16th.	withdrew	75·00
"	19th.	"	12·48
"	26th.	"	72·80
"	30th.	"	9·70

3. Balance Nov. 1st. $172·52

	Nov. 2nd.	withdrew	$ 25·00
	" 9th.	"	15·00
	" 10th.	deposited	31·20
	" 13th.	"	234·50
	" 15th.	withdrew	29·75
	" 25th.	"	110·00
	" 26th.	"	100·00
	" 27th.	"	14·50
	" 28th.	"	10·00

4. Balance Mar. 1st. $251·32

	March 3rd.	withdrew	$ 38·15
	" 5th.	"	20·00
	" 17th.	deposited	90·00
	" 18th.	withdrew	50·00
	" 23rd.	deposited	100·00
	" 24th.	withdrew	75·00
	" 26th.	"	25·00
	" 28th.	"	116·75

THRIFT STAMPS AND DOMINION SAVINGS CERTIFICATES

The Dominion Government has prepared a plan to encourage the boys and girls of Canada to save their earnings and to invest them profitably. The plan is as follows:

The Government sells **thrift stamps** at 25c. each. Seventeen of these are exchanged for a Dominion Government Savings Certificate. At the end of three years from the date of exchange or purchase, the Government will pay $5 for the certificate.

At any time during 1920, a Dominion Savings Certificate of a maturity value of $5 can be purchased for $4·25 in cash. The purchase price and the maturity values of

these certificates, which are of different denominations, are shown below:

Purchase price at any time during 1920	Value three years from date of purchase
$4·25	$5·00
8·50	10·00
21·25	25·00
42·50	50·00
85·00	100·00

The special features of the Dominion Government Dominion Savings Certificates are:

1. Interest is added to the value of these certificates every six months.

2. The certificates will be the same price throughout the year.

3. The certificates can be cashed at any time at any Money-Order Post-office, but no interest will be paid unless they have been held six months from date of purchase.

4. The certificates are automatically registered at Ottawa immediately after they have been purchased.

EXERCISE 81

1. A girl agreed to pick fruit for a farmer at 3c. a box. Her record for one week was as follows:

Mon. 55, Tues. 65, Wed. 75, Thur. 75, Fri. 75, Sat. 80.

What value of Dominion Savings Certificates and Thrift Stamps can she purchase with her savings if she spent $1·75 of her earnings?

2. A boy sold from his garden vegetables to the value of $29·75. If he invested this money in Dominion Savings Certificates, what will be the value of these Certificates when due?

3. Harry and Jack each earned $42·50 during the holidays. Harry with his earnings bought a bicycle for

his own pleasure, but Jack bought Dominion Savings Certificates. If at the end of three years Harry's bicycle is worth $25·00, compare the value at that time of the holiday earnings of these two boys.

4. The weekly family savings for the month of March, 1920, were as follows:

1st. week, $7·53, 2nd. week, $8·11, 3rd. week, $6·37, 4th. week, $9·59, 5th. week, $3·65.

If the monthly savings were invested in Dominion Savings Certificates and Thrift Stamps, how many Dominion Savings Certificates were purchased? What will the value of these Certificates be when due?

How many more Thrift Stamps will be needed to exchange for a $4·25 D. S. Certificate?

EXERCISE 82

FARM PROBLEMS

1. How many 3-gal. cans of water are needed for a garden plot 4 ft. × 12 ft., in order to get the equivalent of rainfall of 1 inch? (1 gal. of water weighs 10 lb., 1 cu. ft. of water weighs 1000 oz.)

2. In the drainage of a 6-acre field, 50 rods of ditch are laid with 6-in. tile costing $60 per M., 20 rods with 4-in. tile costing $36 per M., and 40 rods with 3-in. tile costing $24 per M. If the ditching cost 50c. per rod and a tile is 1 ft. long, find the total cost of the drains.

3. In Question (2), if the yield of barley on the field is raised from 31 bu. per acre to 46 bu. per acre by the drainage, and if barley is worth $1·50 a bu., how much more than the cost of the drainage is made out of the increase in the crop for the year?

4. Find the net gain from the following 10-acre field of wheat:

(a) Costs:

        (i) Ploughing the field 3 times @ $4 an acre each time

(ii) Disking the field twice @ 60c. an acre each
time

(iii) Putting on 120 loads of manure @ 75c. a
load, and spreading it at 10c. a load

(iv) Harrowing the field 3 times @ 30c. an acre
each time

(v) 20 bu. of seed @ $2·15 a bu. $1 for treat-
ing the seed and $7 for sowing it

(vi) Twine $8, cutting $10·00, shocking $8·00,
hauling in $20·00, threshing $30·00.

(b) Receipts:

300 bu. of wheat worth $1·95 a bu., and 12 tons
of straw worth $12 a ton.

5. A flock of 100 hens averaged 125 eggs each a year.
Another flock of 100 hens averaged 88 eggs each a year.
If the average price of eggs for the year was 45c. a doz.,
how much more profitable was the first flock than the
second one?

6. Keep an accurate account of the feed given your flock
of hens for one month, and the number of eggs laid.
At the local price of feed and eggs, find the profit or the
loss on the flock for the period.

7. The following is the milk record of a herd of 8 cows
for the month of June:

No. 1 gives 560 lb. milk testing 4%   butter fat
"   2 "   500 lb.   "   "   3·5%   "   "
"   3 "   720 lb.   "   "   4·25%   "   "
"   4 "   700 lb.   "   "   5·2%   "   "
"   5 "   650 lb.   "   "   3·2%   "   "
"   6 "   575 lb.   "   "   4%   "   "
"   7 "   520 lb.   "   "   3·25%   "   "
"   8 "   750 lb.   "   "   4·2%   "   "

(a) Find the total weight of butter fat produced
during June.

(b) Find the weight of butter testing 85% butter
fat that could be made from the milk.

8. How much more are the returns from a Jersey cow, which gives 550 lb. of milk a month, testing $5\frac{1}{2}\%$ butter fat, than those from a Holstein giving 750 lb. and testing $3\cdot2\%$ of butter fat if butter fat is worth 75c. a lb.?

9. In a dairy where the cream was skimmed from the old-fashioned shallow pans, the skim milk tested $\frac{4}{5}\%$ butter fat. When a separator was used, the skim milk showed only $\frac{4}{100}\%$. Assuming that the annual average weight of skim milk per cow is 7500 lb., find the weight of butter fat saved annually per cow by the use of a separator.

10. From the data in Question (9), estimate the annual gain from the possession of a separator with a herd of 16 cows, if 6 lb. of butter fat make 7 lb. of butter and the butter sells @ 65c. a lb.

11. If a bottle of cream weighing 10 oz. and testing 25% butter fat sells for 20c. and a lb. of butter 84% butter fat sells for 70c., find how much more profitable it is for a dairyman to sell 200 lb. of cream testing 50% butter fat, than it is to sell the butter which he can make from the cream.

12. Keep an accurate account of the costs and the receipts in connection with a field of oats raised on your father's farm. Estimate the profit or the loss from the field.

Commercial Fertilizers

The following is a list of commercial fertilizers, with their composition and price:

(a)	Nitrate of soda	15% nitrogen	$150	ton
(b)	Superphosphate	14% phosphoric acid	42	"
(c)	Dried blood	14% nitrogen	70	"
(d)	Bone meal	22% phosphoric acid	55	"
(e)	Basic slag	12% phosphoric acid	120	"
(f)	Barnyard manure	$\begin{cases} \cdot5\% \text{ nitrogen} \\ \cdot5\% \text{ potash} \\ \cdot5\% \text{ phosphoric acid} \end{cases}$	2	"
(g)	Chloride of potassium	50% potash	150	"
(h)	Ammonium sulphate	20% nitrogen	100	"

Note.—A mixed fertilizer labelled 4, 9, 2, is understood to contain 4% nitrogen, 9% phosphoric acid, 2% potash.

14. How many lb. of nitrogen in a ton of each of the following: Nitrate of soda, ammonium sulphate, dried blood, barnyard manure.

15. How many lb. of phosphoric acid in a ton of superphosphate? in a ton of bone meal? in a ton of basic slag?

16. What weight of nitrate of soda will be needed to provide as much nitrogen as a ton of barnyard manure?

17. At the prices quoted above, find the price of nitrogen a lb. in nitrate of soda, in dried blood, in barnyard manure.

18. What is the composition of a fertilizer labelled 3,10,6? 3,7,8? 0,6,9?

How much nitrogen in a ton of each of these fertilizers?

19. Bordeaux mixture consists of 6 lb. of bluestone, 4 lb. of lime, and 40 gal. of water. Find the cost of spraying a 5-acre field of potatoes five times in a season if it takes 75 gal. of the spray per acre each spraying. Bluestone costs 40c. per lb. and lime 2c. per lb. If the increase in yield from spraying is 60 bu. per acre, find the net gain on the field, reckoning the labour at 50c. per acre per spraying, and the price of potatoes at $3.00 per bushel.

EXERCISE 83

HOUSEHOLD PROBLEMS

1. What is the price paid for 4 chickens weighing as follows: $2\frac{1}{2}$ lb., $2\frac{3}{4}$ lb., $3\frac{5}{8}$ lb., and $3\frac{1}{4}$ lb. @ 28c. a lb.?

2. Find the cost of the family fuel bill for the year 1919:

> 15 tons of furnace coal @ $12·50 a ton
> 500 lb. of cannel coal @ $7·50 a ton
> 2 tons of stove coal @ $12·25 a ton
> 1 cord of kindling wood @ $13 a cord
> 2 cords of wood @ $17 a cord.

3. The tax bills for last year were as follows: Property tax, \$210·75; income tax, \$37·62; business tax, \$162·25. If 5% was allowed off the property and business taxes for paying them promptly, find the amount of the taxes paid.

4. For a certain house lighted by the Hydro current, a flat rate of 60c. a month is charged. In addition, 2½c. a kilowatt hour is charged. If the number of kilowatt hours used in a month is 153, find the light bill for the month if a discount of 10% is allowed for prompt payment.

Write a cheque for the amount. Write a receipt for the bill paid

5. The cost of clothing for a family for 3 months was as follows:

Dressmaking 15 days @ \$2·50 a day, 1 overcoat \$62, 1 suit \$85, less 5% off, 16 yd. of dress goods @ \$1·85 a yd. Miscellaneous items \$35·47. Find the total amount.

6. If the reading of a gas meter on Jan. 5th. was 125280 cu. ft. and on Feb. 5th. 128560 cu. ft., find the cost of the gas for the month @ \$1·35 per thousand cu. ft., less 10% for being paid before Feb. 15th.

Write the cheque for the gas bill.

7. Find the amount of the following family meat bill for one week:

$3\frac{3}{4}$ lb. steak @ 48c.
$5\frac{1}{2}$ lb. rib roast @ 42c.
$3\frac{1}{2}$ lb. pork @ 46c.
2 lb. ham @ 75c.

Write the cheque in payment of the bill.

Write the receipt which the payer should receive.

8. Find the cost of the following furnishings for a living-room: A chesterfield, \$125; a table, \$75; a reading lamp, \$32·50; curtains, \$45; two cosy chairs, \$45 each; 3 other chairs, \$15 each; 3 pictures, \$17·50 each; a bookcase, \$45. A discount of 15% was allowed off for cash.

9. Make out and receipt the following grocery bill: $10\frac{3}{4}$ lb. of butter @ 68c.; 5 bu. 3 pk. of apples @ 75c. a pk.; 8 bu. 1 pk. of potatoes @ 50c. a pk.; $3\frac{1}{4}$ lb. of cheese @ 36c.; $3\frac{1}{2}$ lb. of nuts @ 40c. a lb.

Prices of Food

Butter...60c. a lb.	Milk......12c. a qt.
Flour... · 8c. a lb.	Cream....60c. a qt.
Sugar....15c. a lb.	Eggs......60c. a doz.
Baking-powder 60c. a lb.	

---

2 cups of butter weigh 1 lb.
2 cups of sugar   "   1 lb.
4 cups of flour   "   1 lb.
1 cup = 16 tablespoonfuls.
2 cups of milk = 1 pt.

In $\frac{1}{4}$ lb. of baking-powder there are 30 teaspoonfuls.

---

10. One pint of milk, 1 egg, 1 cup of sugar, and 1 qt. of cream will make 3 qt. of ice-cream. Allowing 15c. for flavouring and ice for freezing, what will the profit be if the ice-cream sells at 10c. a serving? (1 qt. = 9 servings.)

11. What is the cost of a cake made from the following recipe: $\frac{1}{4}$ cup of butter, $\frac{1}{2}$ cup of sugar, 1 egg, $\frac{1}{2}$ cup of milk, $1\frac{1}{2}$ cups of flour, $2\frac{1}{2}$ teaspoonfuls of baking-powder?

12. 1 qt. of milk yields 976 calories or food units, and costs 12c.; 1 lb. of beefsteak yields 1559 calories, and costs 45c. What is the difference in the food value of $1·00 worth of milk and $1·00 worth of steak?

13. The following recipe will make 60 cookies: $\frac{1}{2}$ cup of butter, 1 cup of sugar, 1 egg, $\frac{1}{4}$ cup of milk, $2\frac{1}{2}$ cups of flour, 2 teaspoonfuls of baking-powder. Calculate the cost of 1 doz. cookies.

PRACTICAL MEASUREMENTS

PAVING

The cost of laying concrete floors, walks, and pavements is estimated by the square foot or the square yard.

EXERCISE 84

1. Find the cost of a concrete floor for the cellar of a house 36 ft. long and $28\frac{1}{2}$ ft. wide @ 32c. a sq. ft.

2. What will be the cost of a cement walk 5 ft. 6 in. wide and 90 ft. long @ 39c. a sq. ft.?

3. A corner lot is 66 ft. wide and 125 ft. deep. Find the cost of laying a cement side-walk 5 ft. wide along the front and side of it @ $3·75 a sq. yd.

4. A walk 6 ft. wide and 64 ft. long is made of bricks laid upon the face 2 in. $\times$ 8 in.

If the bricks are worth $17 per M. and the labour is worth $37·50, find the cost of laying the walk.

5. Find the cost of paving a street 38 ft. $\times$ 350 ft. @ $6·75 a sq. yd.

6. A street 41 ft. 4 in. $\times$ 422 ft. is paved @ 75c. a sq. ft. Find the cost of the pavement.

7. A city wishes to put a side-walk 6 ft. wide around the outside of a public square 175 ft. long and 125 ft. wide. How much will it cost @ $5·85 a sq. yd.?

PAPERING

Wall-paper is put up in single rolls 18 inches wide and 8 yards long, and in double rolls 16 yards long. The paper is put upon the walls in strips. Borders are sold by the linear yard.

For rooms not more than 10 feet in height, the following rule is used to approximate to the amount of paper required for the room:

Find the perimeter of the room, less the sum of the widths of the doors and the windows. Allow a single roll for every $3\frac{1}{2}$ feet in this result.

NOTE.—If the room is 12 ft. high, allow 1 roll for every 3 ft.

Since the room is not more than 10 feet in height, the length of each strip of paper will not be more than 8 feet, after we allow for the width of the border at the top of the room and the width of the baseboard at the bottom. From a single roll, therefore, three strips can be cut, each of which is 18 inches wide, making a total width of 54 inches, or $4\frac{1}{2}$ feet. But we have to make allowance for matching and for the spaces to be covered above the doors and windows, and so it is necessary to reduce the width covered by a roll from $4\frac{1}{2}$ feet to $3\frac{1}{2}$ feet.

Example: How many single rolls are required to paper the walls of a room 10 feet high, 16 feet long, and 14 feet wide if the room has 2 doors and 3 windows, each 4 feet wide?

Perimeter of room 2 (16 + 14) ft. = 60 ft.
Width of doors and windows... 20 ft.

Reduced perimeter............. 40 ft.

Number of single rolls = 40 ft. ÷ $3\frac{1}{2}$ ft. = $11\frac{3}{7}$. As a fraction of a roll cannot be purchased, it will be necessary to buy 12 rolls.

EXERCISE 85

1. How many rolls of paper will be required for the walls of a room 15 ft. × 13 ft. and 10 ft. high with 1 door and 3 windows, each 3 ft. 6 in. wide?

2. How many rolls of paper will be required for the walls of a room 17 ft. 6 in. × 14 ft. 6 in. and 10 ft. high with 2 doors and 2 windows, each 3 ft. 6 in. wide?

3. Find the cost of papering the ceiling of a room 14 ft. × 12 ft., if the strips run lengthwise and 6 in. is

wasted in matching each strip except the first one, and the paper is worth 35c. a single roll.

Find the cost of the border @ 18c. a yard.

4. Measure the length, the width, and the height of your school-room, also the width of the openings.

(a) Calculate the number of single rolls required for the walls of the school-room.

(b) Calculate the number of single rolls required for the ceiling of the school-room.

(c) Find the cost of the paper at 55c. a roll and the border at 25c. a yard.

SHINGLING

Shingles are put up in bunches of 250. Four bunches are supposed to cover 100 sq. ft., that is, one *square* of roof.

NOTE.—If an answer comes to, say, 5¼ bunches, make it 6 bunches, as part of a bunch cannot usually be purchased.

EXERCISE 86

1. How many bunches of shingles are required for roofs of the following areas or dimensions:

(a) 1800 sq. ft.?   (c) 80 ft. × 64 ft.?   (e) 84 ft. × 65 ft.?
(b) 1650 sq. ft.?   (d) 72 ft. × 58 ft.?   (f) 78 ft. × 68 ft.?

2. At $2·75 a bunch, how much will the shingles cost to cover a roof with an area of 1960 sq. ft.?

3. If nails are worth 9c. a lb., if the putting on of 1 bunch of shingles is worth 75c., and the shingles are worth $2·50 a bunch, what will be the cost of shingling a roof of two sides, each 40 ft. × 30 ft. if 1¾ lb. of nails are required for each bunch?

4. A roof has two sides, each 60 ft. × 25 ft. What will it cost to shingle this roof if a bunch of shingles is worth $2·40, if 1¾ lb. of nails at 9c. a lb. are required for each bunch, and if the putting on of 1 bunch is worth 65c.?

LATHING AND PLASTERING

In estimating the cost of lathing and plastering, the following quantities of materials are allowed for *100 sq. yd.*:

1400 laths, 10 lb. of nails, 500 lb. of lime, $1\frac{2}{3}$ cu. yd. of sand, and 4 bu. of hair.

If laths are worth $2\frac{1}{2}$c. each, nails 15c. a lb., lime 2c. a lb., sand \$2·00 a cu. yd., hair 75c. a bu., and the labour for each sq. yd., is worth 50c., find the cost of lathing and plastering 100 sq. yd. Find to the nearest cent the cost of lathing and plastering 1 sq. yd.

In plastering, the allowance made for openings is a matter of arrangement with the workman, but it is usual to deduct *one half of the area* of the openings.

EXERCISE 87

1. At \$1·03 a sq. yd. what will it cost to lath and plaster a wall 9 ft. high and 27 ft. long?

2. At \$1·05 a sq. yd. what will it cost to lath and plaster the 4 walls of a room 12 ft. high, 18 ft. long, 16 ft. wide?

3. At \$1·05 a sq. yd. what will it cost to lath and plaster the walls and ceiling of a room 11 ft. high, 15 ft. long, 12 ft. wide?

4. At \$1·05 a sq. yd. what will it cost to lath and plaster the walls and ceiling of a room 12 ft. high, 16 ft. long and 14 ft. wide, if there are 2 doors 7 ft. $\times$ 4 ft. and 3 windows 6 ft. $\times$ $3\frac{1}{2}$ ft. in the room?

(Deduct $\frac{1}{2}$ of the area of the openings.)

5. At 96c. a sq. yd. find the cost of plastering the walls and ceiling of a room 18 ft. long, 14 ft. wide, and 6 ft. above the wainscot; one half the area of one window 3 ft. $\times$ 5 ft. and of one door 7 ft. $\times$ 3 ft. being deducted.

6. Find the cost of painting the walls and the ceiling of your class-room @ 75c. a sq. yd.

7. Measure the dining-room in your home. Estimate the cost of lathing and plastering the walls and the ceiling at $1·05 a sq. yd.

FLOORING

Contractors usually allow $1\frac{1}{4}$ feet, board measure, for every foot of floor to be covered, because there is a loss on account of the tongue and groove matching of the boards. *If the boards are less than 3 in. in width, $1\frac{1}{3}$ board feet for every square foot of floor is allowed.*

Example: How many feet of flooring $2\frac{1}{4}$ in. wide and 1 in. thick will be required for a room 18 ft. × 16 ft.?

The number of square feet in the floor = 18 × 16 = 288.

∴ the number of board feet = 288 × $\frac{4}{3}$ = 384.

EXERCISE 88

1. Find the number of feet of flooring $2\frac{1}{4}$ in. wide needed for rooms of the following dimensions:

(*a*) 18 ft. × 12 ft.

(*b*) 21 ft. × 15 ft.

(*c*) 14 ft. 6 in. × 11 ft. 8 in.

2. Find the number of feet of flooring 3 in. wide needed for rooms of the following dimensions:

(*a*) 13 ft. 6 in. × 12 ft. 4 in.

(*b*) 12 ft. 4 in. × 10 ft. 9 in.

(*c*) 15 ft. 3 in. × 12 ft. 8 in.

3. What is the cost of laying a floor 24 ft. × 18 ft. with maple boards 3 in. wide and 1 in. thick, the labour costing $12·50, and the lumber being worth $125 per M.?

4. What is the cost of laying a floor 25 ft. × 28 ft. with maple boards $2\frac{1}{2}$ in. wide and 1 in. thick if the lumber is worth $135 per M. and the labour costs $22·75?

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FLOOR COVERING

Carpet is usually either 27 in. wide or 1 yd. wide. In estimating the amount needed for a room, allowance must be made for matching the pattern. There will, of course, be no loss on the first strip, as the other strips are made to match with it. Unless otherwise stated, the strips run lengthwise of the room.

Example: How much will it cost to carpet a room 15 ft. × 14 ft., if the carpet is 27 in. wide, worth $3·50 a yd., and 6 in. is allowed for matching on each strip except the first?

> 14 ft. = 168 in.
>
> ∴ no. of strips required = $\dfrac{168 \text{ in.}}{27 \text{ in.}}$ = $6\frac{2}{9}$.
>
> This means 7 strips.   Why?
>
> Length of each of 6 strips is 15 ft. 6 in.
>
> Length of carpet required is 6 × 15 ft. 6 in. + 15 ft.
>   = 108 ft. = 36 yd.
>
> ∴ Cost is 36 × $3·50 = $126.

A floor is very often covered with a rug instead of a carpet laid in strips. The value of the rug is estimated by the number of square yards which it contains. Around the outside of the rug a border is left uncovered, which, if the floor is of hardwood, is generally polished.

Example: A room is 18 ft. × 12 ft. On the floor is a rug which leaves uncovered a border 18 in. wide all around the room. What will it cost to polish the border @ $1·75 a sq. yd.?

> Area of room = 18 × 12 × 1 sq. ft. = 216 sq. ft.
>
> Area of rug = 15 × 9 × 1 sq. ft. = 135 sq. ft.
>
> ∴ area of border = 81 sq. ft. = 9 sq. yd.
>
> Cost of polishing border = 9 × $1·75 = $15·75.

EXERCISE 89

1. Find the cost of covering a porch floor 9 ft. × 30 ft. with plain matting @ $2·25 a sq. yd.

2. The living-room of a summer cottage is 17 ft. × 24 ft., and the floor is covered with plain matting 1 yd. wide laid so as to leave no waste. Find the cost of it @ $1·25 a yd.

3. A floor 18 ft. × 15 ft. is covered with carpet 27 in. wide running lengthwise. How many strips are there? How many yd. long is each strip? How many yd. of carpet are there altogether? What would the carpet cost @ $2·50 a yd.?

4. Measure your school-room and compute the cost of carpeting a room of the same size with carpet 27 in. wide, the waste allowed on each strip except the first one to be 6 in., and the carpet to cost $3 a yard.

5. A room is 36 ft. long and 27 ft. wide. On the floor is a rug which leaves uncovered a border 18 in. wide all around the room. What will it cost to polish the border @ $1·65 a sq. yd.? What will the rug cost at $4·65 a sq. yd.?

6. A room is 15 ft. × 12 ft. What will be the dimensions of a rug for it which leaves uncovered a border 9 in. wide all around the room? What will the rug cost @ $7·50 a sq. yd.? What will it cost to polish the border at $2·75 a sq. yd.?

BRICK WORK AND STONE WORK

Bricks are measured and sold by the thousand. A standard brick is 8 in. long, 4 in. wide, and 2 in. thick, and has a volume of 64 cu. in., or $\frac{1}{27}$ of a cu. ft. When allowance is made for the mortar in a brick wall, it is reckoned that there are 22 bricks in every cubic foot of a wall.

In estimating the cubical contents of a cement wall under a building, the contractor, to get the length

of the wall, measures around the outside of it. He, therefore, counts the corners twice. This is considered fair, because of the extra trouble in laying and shaping the corners.

Under a cement foundation there is always a footing. For ordinary walls 10 in. to 12 in. thick, this footing is 18 in. wide and 6 in. high.

The cost of a cubic yard of cement wall at the present time is made up as follows:

> $1\frac{1}{8}$ cu. yd. of gravel @ $2 a cu. yd
> 2 bbl. of cement @ $2·80 a bbl.
> Labour, $2·00
> Total cost—$9·85 a cu. yd.

Example: Find the number of cu. yd. in the footing for a cement foundation under a building 33 ft. long and 29 ft. wide, if the wall is 1 ft. thick and the footing is 18 in. wide and 6 in. high.

Measuring around the outside of the footing, the length of it will be 2 (33 ft. 6 in.+ 29 ft. 6 in.) = 2 × 63 ft. = 126 ft.

The cubical contents of the footing will be:

$126 \times \frac{18}{12} \times \frac{1}{2} \times \frac{1}{27}$ times 1 cu. yd.= $3\frac{1}{2}$ cu. yd.

EXERCISE 90

1. How many thousands of bricks will be required for a wall 25 ft. long, 20 ft. high, and 2 ft. thick?

2. At $9·85 a cu. yd. what will be the cost of a cement wall 40 ft. long, 1 ft. wide, and 8 ft. high?

3. What would the footing for such a wall be worth if the footing is 40 ft. 6 in. long, 18 in. wide, 6 in. high, and costs $9·85 a cu. yd.?

4. Find the number of cu. yd. of cement in the foundation under a building 30 ft. long and 24 ft. wide if the wall is 1 ft. thick and 8 ft. high and rests on a footing 18 in. wide and 6 in. high.

5. A wall is made up of three sections as follows:

(*a*) A cement footing 100 ft. long, 2 ft. wide, and 10 in. high

(*b*) A cement wall 99 ft. long, 18 in. thick, and 7 ft. high

(*c*) A brick wall 99 ft. long, 12 in. thick, and 15 ft. high.

(i) Find the cost of the footing @ $9·75 a cu. yd.

(ii) Find the cost of the cement wall @ $9·75 a cu. yd.

(iii) Find the cost of the brick wall, allowing 485 cu. ft. for openings if the bricks are worth $25 a thousand and labour for laying one thousand bricks costs $3·50.

(iv) Find the total cost of the wall.

BOARD MEASURE

In estimating the number of board feet in a board lumber-men use a rule of which the following diagram is a representation:—
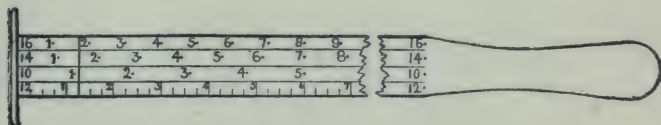
The lower part of the rule is graduated in inches so that the numbers opposite 12 represent inches as well as the number of board feet in boards which are 12 ft. long.

EXERCISE 91

By reading from the rule, find the number of board feet in each of the following boards:

1. $12' \times 7'' \times 1''$ 7. $16' \times 6\frac{3}{4}'' \times 1''$
2. $14' \times 5'' \times 1''$ 8. $10' \times 4\frac{3}{4}'' \times 1''$
3. $16' \times 4'' \times 1''$ 9. $12' \times 3\frac{1}{4}'' \times 3''$
4. $16' \times 3\frac{1}{4}'' \times 1''$ 10. $14' \times 4\frac{3}{4}'' \times 1''$
5. $12' \times 5\frac{1}{2}'' \times 2''$ 11. $16' \times 5\frac{1}{2}'' \times 1''$
6. $14' \times 6\frac{1}{2}'' \times 2''$ 12. $10' \times 4\frac{1}{4}'' \times 1''$

EXERCISE 92 (REVIEW)

1. Add at the same time two columns of the following addition questions.

Check each answer.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
82	76	94	88	94	86	5894
97	48	76	94	78	53	8937
68	93	29	73	62	92	4658
47	78	74	29	99	78	7392
65	69	93	87	87	65	8946
83	47	85	76	46	49	7393
97	58	49	48	39	83	8276
46	73	73	93	87	27	4998
59	99	82	78	49	93	6874

2. Use a short method to do the subtraction work in each of the following questions.

Check each answer.

(a)	(b)	(c)	(d)
3984	2847	47635	58973
288	206	396	1499

3. Use short methods to do the following.
Check each result.

> (a) Multiply $952·75 × 1000
> (b)   "   693 × 99
> (c)   "   88 × 82
> (d)   "   95 × 95

4. Use short methods to find the value of the following.
Check each result.

> (a) 48 tons of coal at $12½ a ton
> (b) 65 head of cattle at $65 each
> (c) 56 yards of cloth at $1·87½ a yard
> (d) 72 doz. pencils at 62½c. a doz.

5. (a) Divide $3865·43 by 1000
> (b)   " $8·64 by 33⅓
> (c)   "   726 by 83⅓
> (d)   " $84·75 by 25

6. Write as decimals:

> (a) Seven-hundredths
> (b) Twelve *and* thirty-six thousandths
> (c) Four hundred and twenty-five *and* nine ten-thousandths
> (d) Four thousand three hundred and sixty-five *and* one hundred and thirteen-thousandths.

7. If the distance from Toronto to North Bay is 227·238 miles, from North Bay to Cobalt is 104·156 miles, and from Cobalt to Cochrane is 149·579 miles, find the distance from Toronto to Cochrane.

8. How many yards are there in 15·75 miles?

9. The value of the English pound sterling was, on a certain date in 1920, because of war conditions, equal to $4·425 in Canadian money.

If a person in Canada, on that date, received a cheque for £75 10s., what would be the value of this cheque in Canadian money?

10. Find the value of 6973 feet of lumber at $75·50 a thousand feet.

11. Express the following percentages as fractions in their lowest terms:

25%, 33⅓%, 14⁴⁄₇%, 37½%, 62½%, 87½%, 83⅓%, 66⅔%.

12. If an article which costs $27·75 has advanced in price to $37·00, to what per cent. increase on the former price is the latter price equal?

13. Find the rate of discount in selling a suit for $59·50 which was marked $68.

14. There are 875 pupils in a school. Find the enrolment if this is 87½% of it.

15. How many Thrift Stamps will $3·75 buy? will $6·25 buy? will $9·50 buy?

16. How many Dominion Savings Certificates and Thrift Stamps will $6·00 buy? will $7·50 buy? will $10·00 buy? will $15·00 buy?

17. What will be the return in 3 years from $85·00 invested now in Dominion Savings Certificates? from $51 invested? from $63·75 invested?

18. Make out a deposit slip for the following deposit. Use any bank you wish. Suppose you are the depositor and J.R. the teller's initials.

4 × $10; 7 × $5; 11 × $2; 27 × $1; coin $11·65; cheques $15·00.

19. J. S. McEwen of Toronto issues a cheque for $75·25 in payment of account to S. M. Ewart. Write the cheque.

Write the endorsement necessary when S. M. Ewart cashes the cheque.

20. Find the minimum monthly balance for the following bank account for the month of March, 1920:

Balance Feb. 29th, $74·86

March	3	Deposited.....$9·89
"	5	" ..... 3·25
"	10	Withdrew..... 5·00
"	20	Deposited..··· 6·19
"	28	Withdrew..... 8·00
"	30	Deposited..··· 5·60

21. A flock of 50 hens are fed daily $7\frac{1}{2}$ lb. of wheat worth $2·00 a bu. If the hens average 90 eggs a year each, and eggs sell at an average of 36c. a doz., what is the year's profit?

22. Make out a market ticket for each of 2 loads of oats if they are worth 95c a bu.:

(a) Gross weight 2900 lb.   (b) Gross weight 2876 lb.
    Wagon   "   1193 lb.   Wagon   "   1235 lb.

23. Find the amount of the following threshing bill:

   2500 bu. of oats @ 3c. a bu.
   1500 bu. of wheat @ 5c. a bu.
    600 bu. of barley @ 4c. a bu.
       4 teams for 3 days @ $5·50 a day for each team
       8 men for 3 days @ $3 a day for each
   2800 lb. of coal @ $9·50 a ton.

24. How much more are the monthly returns from a Jersey cow which gives 650 lb. of milk a month, testing $5\frac{1}{2}\%$ butter fat, than those from a Holstein giving 840 lb. and testing $3·5\%$ of butter fat if butter fat is worth 75c. a lb.?

25. At 30c. a lb., what is the value of the cream containing 30% butter fat taken from 300 lb. milk containing 4% butter fat?

26. (a) Find the number of board feet in a board 14 ft. long, 7 in. wide, and 1 in. thick.

   (b) Compare your result with that obtained from reading the lumber-man's rule.

27. Find the cost of paving a street 220 yd. long and 36 ft. wide @ $7·25 a sq. yd.

28. A room is 32 ft. long and 28 ft. wide.   On the floor is a rug which leaves uncovered a border 18 in. wide all around the room.

(a) What will it cost to polish the border @ $1·50 a sq. yd.?

(b) What will the rug cost @ $4·75 a sq. yd.?

29. How many rolls of paper will be required for the walls of a room 18 ft. by 14 ft. by 10 ft. with 2 doors and 4 windows each 3 ft. 6 in. wide?

30. At 95c. a sq. yd. what will it cost to lath and plaster the 4 walls of a room 10 ft. high, 12 ft. long, and 11 ft. wide, $4\frac{1}{8}$ sq. yd. being allowed for openings?

31. Find the cost of the footing for a cement wall if the footing is 75 ft. long, 18 in. wide, and 6 in. high?

32. Find the cost of a cement wall under a building 28 ft. long and 27 ft. wide if the wall is 7 ft. high and 10 in. thick and costs $9·75 a cu. yd.

(Do not include footing.)

33. Find the cost of the family fuel bill for the year 1920:

$14\frac{1}{2}$ tons of coal @ $13·50 a ton

$3\frac{1}{4}$ tons of stove coal @ $12·80 a ton

3   cords of wood @ $7·75 a cord

1   load of kindling wood @ $9·50.

34. Find the cost of the following furnishings for a living-room:

Table, $65·00; lamp, $37·50; 5 chairs at an average price of $27·50; a book-case, $47·50; and a chesterfield, $135.   A discount of $12\frac{1}{2}\%$ was given for cash.

35. Find the cost of flooring a room 14 ft. by 12 ft. with oak worth $175 per M., allowing $1\frac{1}{4}$ feet, board measure, for every square foot of floor.

SECTION V

Exercise 93 (Review)

1. Write down every factor of: 154, 432, 667, and 945.

2. Find the prime factors of: 684, 841, 1331, 2520, 19635, and 48204.

3. State which of the following numbers are prime and which composite: 117, 351, 269, 1729, and 1367.

4. The product obtained by multiplying a number by itself is 1764. Find the number.

5. Find the prime factors of 3375 and combine them so as to get three equal factors.

6. The product of three consecutive numbers is 4896. Find them.

7. Find the Highest Common Factor of:

> (a) 675 and 1125
> (b) 928 and 1073
> (c) 2037 and 6598
> (d) 390, 910, and 1365
> (e) 432, 720, and 864
> (f) 1404, 1512, and 1638
> (g) 1540, 5005, and 6545.

8. A, B, and C invest $3500 in a business. A invests ¾ as much as B, and C invests as much as A and B together. How much did each invest?

9. What is the length of the longest cord which will exactly measure both the length and the width of a rectangular lot which is 154 yd. long and 132 ft. wide? How many times the length of the cord is the length of the perimeter of the lot?

10. Find the Least Common Multiple of:

(a) 36, 104, and 351
(b) 33, 44, 55, and 132
(c) 119, 136, 252, and 280
(d) 64, 72, 80, 84, and 96
(e) 54, 72, 108, and 168
(f) 403 and 961, 361 and 437
(g) 1403 and 1189.

11. Find the least number which contains 156, 168, 208, and 432 as divisors.

12. What is the lowest number which when divided by 15, by 20, or by 36, will leave a remainder of 7 in each case?

13. If the dividend is $\frac{3}{4}$ of $21\frac{2}{3}$ and the quotient $\frac{2}{3}$ of $6\frac{1}{2}$, what is the divisor?

14. Find the quotients:

(a) $\dfrac{56 \times 27 \times 50}{21 \times 40}$

(b) $\dfrac{25 \times 84 \times 32 \times 36}{30 \times 48 \times 80}$

(c) $\dfrac{69 \times 45 \times 63 \times 54}{46 \times 81 \times 21 \times 27}$

(d) $\dfrac{74 \times 64 \times 57 \times 48}{8 \times 38 \times 96 \times 111}$

15. How many chests of tea, each containing 63 lb. worth 75c. a lb., can be bought for 72 bu. of wheat worth $1·75 a bu.?

16. Find the value of:

(a) $3\frac{10}{11} + 5\frac{7}{15} - 2\frac{9}{22} - 4\frac{9}{10}$
(b) $\frac{11}{17}$ of $21\frac{6}{7} - \frac{11}{19}$ of $10\frac{5}{9} + \frac{11}{21} \div \frac{33}{61}$
(c) $12\frac{13}{30} - 6\frac{8}{15} - 3\frac{33}{50} + 7\frac{41}{75} - \frac{11}{25} + 8\frac{9}{10}$
(d) $5\frac{2}{3} + 3\frac{1}{5} \div 1\frac{1}{3} - 4 \div 1\frac{9}{11}$
(e) $(2\frac{1}{2} - 1\frac{2}{3}) \times (4\frac{1}{3} + 5\frac{4}{9})$
(f) $17 \times 4\frac{5}{11} \times \frac{1}{7}$ of $\frac{2}{28} \times (\frac{3}{28} \div \frac{17}{33})$
(g) $(\frac{5}{18}$ of $1\frac{7}{25} + \frac{8}{9}$ of $1\frac{5}{12}) \div (\frac{4}{9}$ of $2\frac{1}{10} - \frac{5}{8}$ of $\frac{8}{15})$.

17. Find the value of:

    (a) $\dfrac{2\frac{2}{5}}{4\frac{1}{4}}$ (b) $\dfrac{56}{3\frac{3}{8}}$ (c) $\dfrac{4\frac{7}{8}}{52}$ (d) $\dfrac{\frac{3}{8}\frac{3}{3}}{\frac{8}{14}\frac{9}{7}}$.

18. What fraction is:

    (a) 18 of 42?   (c) $3\frac{3}{4}$ of $4\frac{1}{2}$?   (e) $\frac{2}{4}\frac{3}{6}$ of $8\frac{1}{1}\frac{5}{8}$?

    (b) $2\frac{1}{7}$ of 24?   (d) $7\frac{3}{7}$ of $4\frac{8}{11}$?   (f) $\frac{5}{11}$ of $\frac{2}{2}\frac{2}{8}$?

19. The difference between $\frac{2}{3}$ and $\frac{1}{5}$ is what fraction of their sum?

20. (a) 1 bu. 3 pk. is what fraction of 3 bu. 2 pk.?

    (b) 2 qt. 1 pt. is what fraction of 3 gal.?

    (c) 4 gal. 1 pt. is what fraction of 7 gal. 2 qt.?

    (d) 3 yd. 2 ft. is what fraction of 1 rd.?

    (e) £3 2s. 6d. is what fraction of £25?

21. (a) $\frac{2}{4}\frac{9}{7}$ is $\frac{5}{18}$ of what sum?

    (b) $\frac{4}{5}$ of $1\frac{1}{5}$ of \$10 is $4\frac{4}{5}$ of what amount?

    (c) $\frac{2}{3}$ of $22\frac{1}{2}$yd. is $\frac{9}{10}$ of what quantity?

22. If a man pays out $\frac{1}{2}$ of his money, then $\frac{1}{3}$ of what remains, and then $\frac{1}{4}$ of what still remains, what fraction of the whole amount has he now left? If he has \$80 left, how much money had he at first?

23. Divide $\frac{5}{6}$ by $\frac{3}{4}$. If in $\frac{3}{4}$ of a farm there are 120 acres, how many acres are there in $\frac{5}{6}$ of it?

24. Divide $6\frac{2}{3}$ by $3\frac{3}{4}$. If $3\frac{3}{4}$ lb. of tea cost \$1·80, what will $6\frac{2}{3}$ lb. cost?

25. If \$625 is given to two men so that one receives $\frac{2}{3}$ of what the other receives, how much does each get?

26. A man with \$22,500 wills $\frac{1}{3}$ of it to his wife, $\frac{1}{2}$ of it to his son, and the rest of it to his daughter. The wife then wills $\frac{2}{3}$ of her share to the son and the rest to the daughter. How much more will the son have than the daughter?

27. If 16 horses require $8\frac{3}{4}$ bu. of oats for $3\frac{1}{3}$ days, how many bu. will 12 horses require for $5\frac{1}{3}$ days?

28. A merchant bought $16\frac{1}{2}$ yd. of cloth at \$3$\frac{3}{4}$ a yd. and sold it at \$4$\frac{1}{4}$ a yd. What was his gain?

29. A farmer sold the north half of the north-east one-quarter of his 160-acre farm for $3600. What is the remainder of the farm worth at the same rate?

30. The chest expansion of six boys at a certain school was as follows:

$1\frac{3}{4}$ in., $1\frac{5}{8}$ in., $1\frac{7}{10}$ in., $2\frac{2}{5}$ in., $1\frac{1}{2}$ in., $2\frac{1}{4}$ in.

What was the average expansion?

31. A woman who had $240 spent $\frac{1}{5}$ of it for coal, $\frac{1}{2}$ the remainder for rent, and $56 for furniture. What fraction of her money had she left?

32. If to make corn bread for 15 persons it requires $1\frac{1}{2}$ lb. of corn meal, $\frac{3}{4}$ lb. of flour, $\frac{3}{8}$ oz. of sugar, 2 oz. of lard, and 2 oz. of baking-powder, make out a recipe for corn bread for 12 people, and find what it would cost at $2·50 a cwt. for corn meal, $7·50 a cwt. for flour, $6·40 a cwt. for sugar, 32c. a lb. for lard, and 30c. a lb. for baking-powder.

33. A man who has a sum of money on deposit in a bank draws out $\frac{8}{9}$ of it. With $\frac{3}{4}$ of this sum he buys a house, and with the remainder, $1800, he buys an automobile. How much money has he still in the bank?

DECIMALS

EXERCISE 94 (REVIEW)

1. Add:

        (a) 3·405, 50·76, 301·632, 90·09, and 1·003

        (b) 43·587, ·6093, ·0008, 132·9, and 406.

2. Subtract:

        (a) 4·637 from 19·859

        (b) 17·1825 from 210

        (c) 13·60411 from 34·012.

3. Find the value of:

        (a) 421·37 − 9·085 + ·815 − 63·2

        (b) 27 − 11·26 + 3·842 + 79·0483

        (c) 105·47 + 39·748 − 23·1097 − 84.

4. Multiply:

     (a) 83·04 by ·0625
     (b) 57·3246 by ·0065
     (c) 67 by 4·819
     (d) 23·57 by 3·048.

DIVISION OF DECIMALS

In the division of integers by integers, how is the first digit in the quotient obtained? How may its place value be known?

If 65879 is divided by 53, what is the first digit in the quotient? What is its place value? What, then, must be the place value of the second digit in the quotient? of the third? Again, if 658·79 be divided by 53, what will the first digit in the quotient be? What is its place value? What is the place value of the second digit? of the third? of the fourth?

658·79 may, therefore, be divided by 53 as follows:

$$
\begin{array}{r}
12\cdot43 \\
53)\overline{658\cdot79} \\
53 \\
\hline
128 \\
106 \\
\hline
227 \\
212 \\
\hline
159 \\
159 \\
\end{array}
$$

If the divisor had been 5·3 or ·53, how could it be changed to the integer 53? What change must then be made in the dividend to leave the quotient unchanged?

It is clear, therefore, that in the division of decimals the divisor can always be made a whole number. For instance, $16\cdot37 \div 4\cdot2 = 163\cdot7 \div 42$ and $\cdot00378 \div \cdot64 = \cdot378 \div 64$. Hence the division of decimals can always be performed as in the foregoing Example. It will be seen that in dividing by a whole number the decimal point in the quotient comes immediately above that in the dividend.

<div align="center">EXERCISE 95</div>

1. Divide:

   (a) $87\cdot5$ by 35 (f) $55\cdot1862$ by 387
   (b) $32\cdot68$ by 19 (g) $1\cdot10166$ by 427
   (c) $\cdot1518$ by 46 (h) $286\cdot9061$ by 83
   (d) $7\cdot504$ by 56 (i) $\cdot00427638$ by 789
   (e) $\cdot03381$ by 147 (j) $6941\cdot902$ by 2134.

2. Divide:

   (a) 35209 by $25\cdot7$ (e) $\cdot348336$ by $\cdot492$
   (b) 5658 by $\cdot0123$ (f) 24 by $19\cdot2$
   (c) $59\cdot4204$ by $5\cdot86$ (g) $8\cdot31183$ by $23\cdot05$
   (d) $\cdot461071$ by $12\cdot23$ (h) $2\cdot5346304$ by $4\cdot632$.

3. Divide $437\cdot25$ by 10, by 100, by 1000, by 10000. What change is made in the position of the decimal point by each of these divisions?

4. What is the quotient when 4368 is divided by 100? When $37\cdot5$ is divided by 10000?

5. Divide 4 by 5, 3 by 60, 8 by 500, 7 by 8, 1 by 16, 2 by 25, 9 by 20.

6. Divide, to four places of decimals:

   (a) $64\cdot375$ by $9\cdot573$ (d) 3 by 7
   (b) $4\cdot50775$ by $123\cdot5$ (e) 1 by 13
   (c) 6 by 17 (f) 14 by $1\cdot9$.

7. Divide:

(a) ·35209 by 25·7
(b) 15·812 by 23·6
(c) 8·547 by ·077.

8. Divide, to three places of decimals:

(a) 64·375 by 9·573
(b) 759·56 by 43·01.

9. Simplify, using cancellation where possible:

(a) $\dfrac{7\cdot 2 \times 8\cdot 4}{5\cdot 6}$

(b) ·1 × ·02 × 2·3 × ·007

(c) $\dfrac{630}{\cdot 35} \div \dfrac{700}{1\cdot 75}$

(d) $\dfrac{\cdot 12}{\cdot 049} \times \dfrac{\cdot 28}{\cdot 0045} \times \dfrac{\cdot 0035 \times 18}{\cdot 96}$.

10. Divide 15·04 by 47 and then write down the value of 150·4 ÷ 4·7 and of 1·504 ÷ ·47.

11. If a cwt. of sugar costs \$16·75, how many lb. can be purchased for \$2261·25?

12. How many bu. of wheat at \$1·95 a bu. must a man sell to pay a debt of \$637·65?

13. A train runs 49·32875 mi. in 1·273 hr. What is the rate in miles per hour?

14. How many cwt. of flour are there in 3625 lb.? What is the flour worth at \$13·65 a cwt.?

15. Find the value of 4357 board feet of lumber @ \$45·50 a thousand feet.

16. Find the value of 625 cu. ft. of gas @ \$1·35 a thousand cu. ft.

17. In the Imperial gallon there are 277·274 cu. in. Find to two decimal places the number of gallons of water in a rectangular tank 3 ft. wide, 4 ft. deep, and 9 ft. long if the tank is full.

18. The total attendance at a certain school for the month of March, in which there were 23 teaching days, was 652. Find to two places of decimals the average daily attendance.

19. A base-ball team played 78 games, of which it won 53. Find to three places of decimals its winning average.

20. What is the width of a sheet of paper the length of which is 12·5 in., and which contains 92·5 sq. in.?

21. A dealer receives a shipment of stoves, the total weight being 9·75 thousand pounds. He pays $70·20 freight. What is the freight per thousand?

TABLES

A very practical application of decimals will be found in the construction of tables for making rapid calculations.

For example, the treasurer of a Cheese Company, by comparing the number of pounds of milk used with the money received for the cheese made, is able to estimate the value of 1 lb. of milk, and from this can make the following table:

No. of lb.	Value	No. of lb.	Value	No. of lb.	Value
1	$·025362	4	$·101448	7	$·177534
2	·050724	5	·126810	8	·202896
3	·076086	6	·152172	9	·228259

If, now, he wishes to find the value of 12375 lb. of milk, he may proceed thus:

10000 lb. have a value of	$253·62 (read from table)
2000 lb. " " " "	50·72
300 lb. " " " "	7·60
70 lb. " " " "	1·77
5 lb. " " " "	·12
12375 lb. " " " "	$313·83.

What is the practical value of such a table?

EXERCISE 96

1. The Cook Cheese Manufacturing Company, during the month of June, 1919, used 614095 lb. of milk in making cheese, which was sold for $16763·82. If the month's expenses amounted to $2193·08—

(a) Find in dollars to 6 places of decimals the price which they could pay for a pound of milk.

(b) From the value of 1 lb., find the value of 2, 3, 4, 5, 6, 7, 8, and 9 lb.

2. Find, from the results in (a) and (b) above, the amounts of the cheques which the patrons who sent the following weights of milk for the month should receive:

(a) 8456 lb.	(c) 11482 lb.	(e) 27893 lb.
(b) 9473 lb.	(d) 13562 lb.	(f) 34265 lb.

3. The Riverside Cheese Manufacturing Co., during the first two weeks in September, 1919, used 174232 lb. of milk in making cheese, which was sold for $3730·97. If the expenses for the two weeks amounted to $275·33 and the net proceeds are divided among the patrons according to the amount of milk each of them sends to the factory, find in dollars to six places of decimals:

(a) How much can be paid for 1 lb. of milk.

(b) How much 2, 3, 4, 5......9 lb. are worth.

(c) What the patrons should receive for the following weights of milk:

(i) 8427 lb.   (ii) 11468 lb.   (iii) 13467 lb.

4. A Farmers' Co-operative Society sold a load of hogs weighing 30653 lb. in Toronto and received for the load $5670·81.

(a) If the expenses in connection with the shipment amounted to $89·65, find in dollars to six places of decimals the price per lb. received.

(b) Make out a table from which the amount due to farmers whose hogs weighed 1587 lb., 3542 lb., 6847 lb., 1242 lb., 5492 lb., can be easily calculated.

REDUCTION OF DECIMALS TO VULGAR FRACTIONS

Example: Express ·32 as a vulgar fraction:

$$·32 = 32 \text{ hundredths} = \tfrac{32}{100} = \tfrac{8}{25}.$$
$$\text{Again } 6·32 = \tfrac{632}{100} = 6\tfrac{32}{100} = 6\tfrac{8}{25}.$$

EXERCISE 97

Express as vulgar fractions in their lowest terms:

1. ·3, ·6, ·8, ·37, ·45, ·64
2. ·05, ·025, ·001, ·675, ·317, ·144
3. ·7375, ·0112, ·0175, ·0674, ·3225
4. 4·75, 10·25, 12·064, 7·275, 15·12.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS

Regarding a vulgar fraction as the quotient obtained by dividing the numerator by the denominator, $\tfrac{3}{4} = 3 \div 4$.

To express $\tfrac{3}{4}$ as a decimal, divide $3·00$ by 4 and the result is ·75.

Example: Reduce $\tfrac{3}{7}$ to 4 places of decimals:

$$\tfrac{3}{7} = 3 \div 7 = ·4285.$$

EXERCISE 98

1. Doing the work mentally, reduce to decimals:

$\tfrac{1}{2}$, $\tfrac{1}{4}$, $\tfrac{3}{4}$, $\tfrac{1}{5}$, $\tfrac{2}{5}$, $\tfrac{3}{5}$, $\tfrac{4}{5}$, $\tfrac{1}{8}$, $\tfrac{3}{8}$, $\tfrac{5}{8}$, $\tfrac{7}{8}$, $\tfrac{1}{10}$, $\tfrac{3}{10}$, $\tfrac{7}{10}$, $\tfrac{9}{10}$, $\tfrac{1}{20}$, $\tfrac{3}{20}$, $\tfrac{7}{20}$, $\tfrac{9}{20}$, $\tfrac{1}{25}$, $\tfrac{2}{25}$, $\tfrac{3}{25}$, $\tfrac{4}{25}$, $\tfrac{6}{25}$, $\tfrac{7}{25}$, $\tfrac{8}{25}$, $\tfrac{9}{25}$.

2. Reduce to decimals:

$\tfrac{3}{16}$, $\tfrac{7}{16}$, $\tfrac{17}{25}$, $\tfrac{9}{32}$, $\tfrac{15}{64}$, $\tfrac{23}{25}$, $\tfrac{69}{125}$, $\tfrac{7}{625}$, $\tfrac{17}{40}$, $\tfrac{39}{80}$, $\tfrac{57}{250}$.

3. Reduce to four places of decimals:

$\tfrac{2}{3}$, $\tfrac{5}{6}$, $\tfrac{7}{11}$, $\tfrac{8}{13}$, $\tfrac{7}{15}$, $\tfrac{14}{17}$, $\tfrac{11}{18}$, $\tfrac{16}{23}$, $\tfrac{9}{14}$, $\tfrac{61}{112}$.

4. Find to four places of decimals the value of:

$$\tfrac{5}{12} + ·173 - \tfrac{1}{7} + ·9624 - \tfrac{18}{23}.$$

5. Find to the nearest cent the value of:

$$\$\tfrac{3}{5} + \$·55 + \$\tfrac{3}{8} - \$1·38 + \$\tfrac{5}{11}.$$

6. A man who owns ·625 of a stock of goods sells ¾ of his share. What decimal of the stock has he left? What will be the value of what he now owns if the whole stock is worth $6400?

7. If a work day is 12 hr., what fraction of a work day is 10 hr. 30 min.? To what decimal is this fraction equal? At $2·40 a day of 12 hr., how much would a man earn in 10 hr. 30 min.?

8. What decimal of 1 bu. is 2 pk. 3 qt. 1 pt.? What will 2 pk. 3 qt. 1 pt. of chestnuts be worth at $3·20 a bushel?

9. What decimal of 11 lb. 9 oz. is 2 lb. 5 oz.? If 11 lb. 9 oz. of butter are worth $3·70, what will be the value of 2 lb. 5 oz.?

10. Express 2½ qt. as a decimal of 1 gal. What will 3 gal. 2½ qt. of wine cost at $4 a gal.?

<center>EXERCISE 99</center>

1. A cubic foot of water weighs 10 lb. If cast-iron is 7·2 times as heavy as water, how many cubic feet of cast-iron will weigh as much as 3060 cu. ft. of water?

2. Find the number of pounds in ·175 of a ton + ·235 of a hundredweight + ·35 of a pound.

3. Add together ·029 of 1 da. 3 hr., ·45 of 11 hr. 10 min., and ·89 of an hour, and then express the answer in minutes and the decimal of a minute.

4. Express, in inches, the difference between ¾ yd. and ·875 ft.

5. A base-ball player's "batting average" is calculated by finding what decimal the number of his "safe hits" is of the number of times he was "at bat." Find the batting average, to three decimal places, of the player who, out of 59 times "at bat," made 18 "safe hits."

6. A cricketer made 1148 runs in 68 innings. Calculate to 2 decimal places the average number of runs an innings.

7. Find to two decimal places the average weight of 7 boys whose individual weights were: 88 lb., 95 lb., 93 lb., 110 lb., 102 lb., 87 lb., and 101 lb.

8. Divide $35·36 among 4 men and 3 boys, giving to each boy ·14 of a man's share.

9. Find the cost of 6250 laths at 32 cents a hundred.

10. Out of 100 parts of rice ·3 parts are fat, ·4 parts are ash, 7·8 parts are proteid, and 79·2 parts are starch. The rest is water. How much water will there be in 60 lb. of rice?

11. At a certain mine a ton of iron ore yields ·55 tons of pure iron. How much pure iron will there be in 84·4 tons of ore?

12. A gentleman finds that he spends ·15 of his salary for insurance, ·48 for living expenses, and ·10 for charity. He invests the remainder, $810. Find his total salary.

13. A railway time-table shows Kenora 1289·4 miles and Winnipeg 1414·9 miles west of Montreal. An express leaves Kenora at 5·30 a.m. and arrives at Winnipeg at 9·45 a.m. Find the average rate of the express in miles an hour.

14. Find the value of: $·3 \times ·03 - ·01 \times ·002 \times 10·4$.

15. By cancellation find the value of: $6·6 \times 2·24 \times 1·56 \times 1·08$ divided by $3 \times ·44 \times 2·8 \times ·81$.

16. By cancellation find the value of: $11·52 \times 1·04 \times 3·36$ divided by $·48 \times 2·6 \times 76·8$.

17. A farmer sowed 2·5 bu. of oats an acre on 10·25 acres, which yielded a crop averaging 38·4 bu. an acre. If his seed grain cost him $·56 a bu. and his other expenses amounted to $3·60 an acre, how much will he gain by selling his oats at $·48 a bu.?

18. An adult who requires only ·25 lb. of meat a day actually consumes 193·5 lb. a year. How much money might he have saved if meat is worth $·37 a lb.?

19. The average temperature of the Province of Ontario for each month of 1918 was as follows:

8·5°, 16·5°, 31·1°, 42·7°, 57·2°, 61·2°, 68·2°, 69·4°, 54·3°, 49·9°, 39·3° and 27·7°. Find the annual mean temperature.

20. The average rainfall (in inches) for the Province of Ontario for each month of 1918 was as follows:

0·02, 1·50, 0·54, 1·38, 2·62, 2·69, 2·18, 2·94, 4·31, 3·18, 2·03, and 1·37. Find the average for the year.

21. A steam pump at each stroke pumps 3·49 gal. of water and makes 51·4 strokes a minute. How many gal. of water will it pump in a day?

BILLS AND ACCOUNTS

(See pages 52, 55, 83)

EXERCISE 100

1. Write in correct form the following account:

John White Lumber Co., Kingston, sold to Samuel Rodd, Portsmouth, lumber as follows:

May 3rd.—1225 ft. spruce @ $35 per M., 2360 ft. maple @ $72 per M., 1800 ft. hemlock @ $37·50 per M.; May 5th.—1710 ft. oak flooring @ $125 per M., 1600 ft. pine @ $62·25 per M.; May 12th.—2500 ft. maple @ $72 per M., 1240 ft. hemlock @ $37·50 per M. Mr. Rodd paid by cheque May 5th, $275, May 12th, $325, and the balance on May 31st. Make out the cheques on the Bank of Montreal, Kingston.

2. On June 10th, 1920, Miss Emily Rose bought of Geo. Fair & Co., Dry-goods Merchants, Belleville:

3¾ yd. lining @ 25½c., 2¾ yd. insertion @ 19½c., 3½ yd. cambric @ 12½c., 2¼ yd. silk @ $1·87½, 4¼ yd. lace @ 35c., ⅚ doz. spools thread @ 50c. a doz., 3½ doz. buttons @ $2·00 a gross. Write out a receipted bill for the above.

3. Make out a cash account for the month of September, 1920, showing receipts, expenditures, and balances for the following:

Sept. 1st.—Have on hand $25·60; Sept. 3rd.—Paid for clothing $8·25; Sept. 5th.—Paid church dues $2·50; Sept. 9th.—Paid for shoes $6·50; Sept. 11th.—Paid for books $2·35; Received wages $36·00; Paid for laundry $2·40; Sept. 18th.—Paid ticket to Toronto $4·85; Paid ticket to theatre $1·50; Sept. 19th.—Paid hotel bill $4·50; Sept. 20th.—Paid car tickets $1·00; Sept. 25th.— Received wages $36·00; Paid laundry $3·10; Sept. 26th.— Paid church dues $3·75; Sept. 28th.—Paid for music lessons $6·00; Sept. 30th.—Paid board $30·00.

4. Make out a statement, showing the receipts and expenses, for the month of September for the following data: A milk dealer has 60 cows, each of which produces on an average 16 qt. of milk daily. He delivers the milk for 15c. a qt., having two men for this purpose at $2·75 each a day. His expenses for feed, horse-hire, rent, and such like, average $1·75 a day for each cow.

5. Make out a bill, supplying names and dates, for the following supplies for a banquet given by the Parents' Association of Queen Mary School: 6 cans of tomatoes @ 20c.; 18 qt. of milk @ 14c., $5\frac{1}{4}$ lb. of flour at $7·50 a cwt.; $2\frac{3}{4}$ lb. of butter @ 62c.; 12 loaves of bread @ 22c.; 6 bunches of celery @ 8c.; 2 lb. of salt @ 8c.; 4 lb. of salmon @ 22c.; 33 eggs @ 52c. a doz.; $12\frac{1}{2}$ lb. of sugar @ 8 lb. for $1·00; $8\frac{1}{4}$ lb. of beef @ 32c.; $2\frac{1}{4}$ pk. of potatoes @ $1·25 a bu.; 8 cans of peas @ $2·40 a doz.; $2\frac{1}{4}$ lb. of coffee @ 52c. a lb.; 10 grape fruit @ 4 for 25c.; 3 bottles of olives @ 30c.

6. Make out a statement, supplying dates and names where necessary, to show the result of the following transactions: On April 1st, 1919, Fred Jones started a cartage business and bought on that date 2 horses for $165 each; 2 sets of harness at $48·75 each; 2 wagons at $95 each; and paid for other necessaries $125. He paid two drivers $65 each a month; paid $18 a month for feed for each horse; and $34 a month for rent, repairs, and incidentals. His wagons are employed 26 days each month, and from the first wagon his receipts are $7·75 a day and from the second $9·25 a day. On June 1st, 1920, he sells out, receiving for the business and outfit $725.

7. Make out a statement, showing the profit or loss on a school-garden plot, from the following particulars:

The ground was ploughed for $2·00. It required 125 lb. fertilizer @ $32 a ton; 3 oz. of beets @ 8c. an oz.; 2 qt. of peas @ 80c. a bu.; 3 pt. of beans @ $3·60 a bu.; 4 packages of lettuce @ 10c. a package; 3½ qt. of corn @ $1·60 a bu.

The yield was 96 bunches of beets @ 5c.; 2 bu. of peas @ 45c. a pk.; 2½ bu. of beans @ 7c. a qt.; 210 head of lettuce @ 6 for 25c.; and 1500 ears of corn @ 20c. a doz.

8. Make up a bank account, showing deposits, withdrawals, and balances, for the following entries:

July 1st, 1920—Balance on hand $128·75; July 3rd.—Deposited a cheque from A. E. Miller $37·95; July 5th.—Withdrew by cheque to R. Hicks & Son $62·19; July 8th.—Deposited salary cheque $175·00; July 10th.—Deposited in cash $148·64; July 14th.—Withdrew by cheque to S. Mills $16·29; July 17th.—Withdrew in cash $95; July 21st.—Withdrew in cash $17·50; July 26th.—Deposited cheque from T. Martin $54·36; July 28th.—Withdrew in cash $28·75.

9. Make out the cheques from A. E. Miller and to R. Hicks & Son.

10. Make out a deposit slip for the $148·64 cash entry—if there were 9 two-dollar bills, 15 ones, 4 tens, the rest being in fives and coins.

11. A doctor charges patients $5 for a first examination, $2 for each house visit, and $1 for each office treatment. Make out a doctor's bill, supplying names, for a patient who was examined, visited at his house 8 times, treated at the office 7 times, and received $6 worth of medicine.

PERCENTAGE

EXERCISE 101

1. Express in vulgar fractions: 175%; $43\frac{3}{4}\%$; $7\cdot5\%$; 96%; $\frac{2}{3}\%$; $3\frac{1}{8}\%$; $\cdot725\%$.

2. Express as a per cent.: $\frac{11}{16}$; $\frac{2}{125}$; $\frac{3}{40}$; $\frac{7}{8}$; $\frac{3}{850}$; $\frac{7}{325}$; $\frac{311}{818}$.

3. What is 140% of $\$8\cdot25$? $58\frac{1}{3}\%$ of £14 8s.? $\frac{4}{7}\%$ of $6\cdot28$? $\cdot55\%$ of $19\frac{1}{11}$? $21\frac{3}{7}\%$ of 105?

4. What is the number or quantity of which $4\cdot14$ is 9%? 35 ft. 9 in. is 75%? $\$2\cdot48$ is 62%? $\frac{4}{7}$ is $62\frac{1}{2}\%$? $\frac{8}{225}$ is $\frac{4}{5}\%$?

5. What per cent. is 2 ft. 8 in. of 8 yd.? 28 pk. 1 qt. of 9 pk. 3 qt.? $\$\frac{3}{5}$ of $\$8\cdot40$? $6\cdot25$ of $37\cdot5$? $4\frac{1}{6}$ of $13\frac{1}{3}$?

6. A merchant sells goods valued at $450, but is able to collect only 80% of the amount. Find the amount collected.

7. At a certain examination the total marks given were 4200. How many marks did a student receive who got $65\frac{1}{2}\%$ of the total?

8. The population of a town is found to have increased 20% every 10 years. Its population in 1891 was 6250. What was its population in 1901? in 1911?

9. The average daily attendance at a school for the month of October was 92% of the number on the roll. If the number on the roll was 325, find the average attendance.

10. A casting which weighs 480 lb. contains $96\frac{1}{2}\%$ of copper, and the rest is tin. How many pounds of tin are there?

11. A man receiving a salary of $1200 a year spends 14% of it for board, 10% of it for clothing, and $13\frac{1}{2}\%$ of it for other expenses. How much can he save in 6 years?

12. A library contains 4800 volumes, of which 1440 are fiction. What fraction of the library is fiction? What per cent.?

13. Of 8280 candidates who wrote on a certain examination 3925 passed. What per cent. failed?

14. To 63 gal. of wine there are added 12 gal. of water. What per cent. of the mixture is the water?

15. Of 51,000,000 bu. of wheat sent out of Canada, all but 850,000 was sent to Great Britain. What per cent. of the wheat exported did Great Britain receive?

16. Cloth when sponged shrinks $1\frac{7}{8}\%$ of its length. It is found that a piece of cloth after sponging measures 13 yd. 3 in. What was its original length?

17. A man's salary is $3250 and his expenses are $2340. What per cent. of his salary does he save?

18. (a) If in a Base-ball League a Toronto team wins 84 games and loses 51, while at the same time a Buffalo team wins 82 games and loses 54, what is the respective standing of the two teams?

(b) If each team has still 8 games to play, and Toronto wins 6 and loses 2, while Buffalo wins 4 and loses 4, what will be their final standing?

19. Sulphate of ammonia contains 25% ammonia, acid phosphate contains 14% phosphoric acid, and muriate of potash contains 50% potash. By mixing certain quantities of these three substances with sand, a ton of a potato fertilizer is produced. If this fertilizer contains 4% ammonia, 8% phosphoric acid, and 4% potash, how many pounds of each of the substances named above did it take to make the mixture?

20. The food value of a quart of milk is equal to that of $\frac{3}{4}$ lb. of beef, or to that of 8 eggs. If milk sells at 15c. a quart, beef at 36c. a lb., and eggs at 54c. a doz., what per cent. is saved by buying milk instead of beef? milk instead of eggs? beef instead of eggs?

21. Three cows produce, respectively, 20, 23, and 27 lb. of milk a day. The first supply contains $4 \cdot 6\%$ of butter fat, the second 5%, and the third $4 \cdot 5\%$. If all the milk is mixed together, what will be the percentage of butter fat in the mixture?

22. (*a*) The Factory Inspection Bureau for Ontario reports that of the 4900 industrial accidents in 1918, 2% were caused by saws, 4% by burns, 5% by scalds, 6% by falling bodies, 8% by shafting, 9% by explosives, 10% by electricity, 12% by elevators, and 15% by machines. What was the number of accidents in each class?

(*b*) If 25% of the above accidents were due to the carelessness of the workmen themselves, how many accidents might have been avoided by reasonable care?

PROFIT AND LOSS

Practically all business transactions are carried on directly or indirectly for **profit,** or to prevent or reduce **loss.** The business man is continually inquiring of himself—"Does my business pay?" Percentage is found a convenient means for making comparisons and measuring results between income and outlay.

Profit and *Loss*, unless otherwise stated, is always expressed as a per cent. of cost.

EXERCISE 102

1. An article which cost $3·60 was sold for $4·20. What fraction of the cost was the gain? What per cent.?

2. I buy a horse for $150 and sell it so as to gain $30. For what per cent. of the cost do I sell it?

3. The cost of a farm was $4500, and the selling price was $4960. What was the gain per cent.?

4. By selling a carriage for $138, a man makes a profit of 15%. What did it cost him? At what should he sell it to make 33⅓%? What fraction of the cost is the selling price in each case?

5. Goods damaged by fire were sold at a loss of 16%. If the total loss was $3600, find the original cost of the goods.

6. A grocer sold oranges for $16·50, gaining thereby 10%. What would have been his loss per cent. had he sold them for $12·75?

7. A tea merchant mixes 40 lb. of tea at 45c. per lb. with 20 lb. at 30c. per lb. What is his gain per cent. by selling the mixture at 50c. per lb.?

8. If $16\frac{2}{3}\%$ is gained by selling a sewing-machine for $42, for what should it be sold to gain $33\frac{1}{3}\%$?

9. A sold goods to B at a profit of 10%, and B sells them to C at a profit of 15%. If C paid $506 for the goods, what did they cost A?

10. A merchant sold goods at a profit of 30% and cleared $385·35. What was the selling price of the goods?

11. A dealer bought 180 bbl. of flour at $7·20 a bbl. He sold 60 bbl. at a loss of $8\frac{1}{3}\%$. At what price a barrel must he sell the remainder so as to gain 20% on the entire transaction?

12. If $29\frac{1}{8}\%$ is lost by selling goods for $137·70, what per cent. would be gained by selling them for $267·30?

13. A grocer mixes 12 lb. of tea worth 75c. a lb. with 15 lb. worth 84c. a lb. At what price a lb. must the mixture be sold to yield a profit of 20%?

14. For making a hat a milliner's expenses were as follows:

$1\frac{1}{2}$ yd. velvet, @ $1·50 a yd.
$\frac{3}{4}$ yd. fur band, @ $4·00 a yd.
1 ornament, @ $3·00
hat frame, 50c.
wire, 15c.
lining, 35c.
labour, $2·75.

If she sells the hat for $16, what per cent. of profit does she make? What per cent. of selling price?

15. A housewife buys apples at 50c. a peck. What per cent. could she save by buying them at $2·75 a barrel if a barrel contains $2\frac{1}{2}$ bu.?

COMMISSION

Commission is the amount received by an agent, factor, collector, or commission merchant for transacting business for his employer or principal.

It is usually a certain per cent. of the *amount received* when selling or of the *amount paid* when buying goods.

The sum which remains after paying commission and other charges is called the **net** proceeds.

EXERCISE 103

1. For collecting accounts an agent receives a commission of 5%. What will be the amount of his commission for collecting $750?

2. A book agent receives 25% on his sales. What will be his commission for selling 6 sets of books at $12·50 a set?

3. An agent sells 450 bbl. of apples at $4 a bbl. on a commission of $3\frac{1}{4}$%. How much will his principal receive from the sales?

4. How much shall I have to send my agent in order that he may purchase for me 600 bu. of oats, at 35c. a bushel, after deducting his commission of $4\frac{1}{2}$%?

5. For renting a house a real estate agent charges 2% of the annual rent. What will be his commission for renting a house at $18 a month?

6. For selling $6200 worth of goods an agent receives a commission of $155. What was the rate charged?

7. A commission merchant bought goods worth $3650 and received $54·75 as his commission. What was the rate charged?

8. A dealer sent his agent $1842·75, instructing him to retain his commission and purchase goods with the balance. The agent's commission was $22·75. What was the rate charged?

NOTE—It is customary to deduct the commission for buying from the money received, and to invest the balance.

9. For selling goods on a commission of $3\frac{1}{2}\%$ an agent received \$39·90. For what amount did the goods sell? How much did the agent's principal realize from the sale?

10. An agent received \$24·80 for purchasing goods on a commission of $2\frac{2}{3}\%$. What was the value of the goods bought? How much did they cost the agent's employer?

11. A canvasser for a newspaper is allowed $22\frac{1}{2}\%$ of all subscriptions received by him. How many subscriptions at \$4 each must he secure in order to earn \$135 commission?

12. A machine agent sold twelve binders on a commission of 15%. If his employer received, as net proceeds, the sum of \$1632, what was the selling price of each binder?

13. An agent remits to his principal \$2488·50 as the net proceeds from the sale of 2800 bu. of wheat. Find the agent's commission if he charges at the rate of $1\frac{1}{4}\%$. Find also the selling price of the wheat a bushel.

14. An agent bought for his employer 20000 bu. of corn at 58c. a bushel and sold it next day at 60c. Find his total commission if he charges $\frac{1}{2}\%$ for buying and also $\frac{1}{2}\%$ for selling. Find also his employer's net gain.

15. An agent sold 782 cwt. of flour on a commission of $2\frac{1}{2}\%$. If he paid \$97·75 for freight and remitted his employer \$2189·60, find the selling price of the flour.

16. I send my agent \$2790 with which to buy cotton at 9c. a yard, after retaining his commission of $3\frac{1}{3}\%$. How many yards of cotton did he buy? What was his commission? What fraction of the cost of the cotton was the money sent?

17. A commercial traveller was offered: (a) a commission of $7\frac{1}{2}\%$ on his sales, or (b) a salary of \$30 a week with a 3% commission on his sales. He accepted the first offer, which he found to be \$1140 a year better than the second. What amount of goods did he sell?

18. A man having bought 3000 lb. of tea, instructs his agent to sell it at 51c. a pound and invest the net proceeds in sugar at 5c. a pound. How many pounds of

sugar were bought if the agent's commissions for buying and for selling were each 2%? What was the agent's total commission?

19. An article which cost $10·80 to manufacture is sold at a price to yield the manufacturer 20% profit after allowing his agents 25% commission. At what price do the agents sell the article?

20. An agent sold some goods for his employer. He retained $3\frac{1}{2}$% commission, paid $16·80 for freight, $9·75 for storage, and remitted his employer $1594·65. Find for what amount the goods were sold.

21. A farmer having disposed of his farm, sells by auction his stock, produce, and farm implements. He employs an auctioneer, who charges him 5% on the first $250 and $2\frac{1}{2}$% on all sales after that. The sales were: 3 horses at $135 each; 6 cows at $87·50 each; 12 young cattle at $40·25 each; 9 pigs at $28·75 each; 25 hens at 65c. each; 1 wagon $35; 2 ploughs at $7·25 each; 1 harrow $14; 1 hay rake $24; and 15 tons of hay at $16·25 a ton. What did the sale net the farmer?

TRADE DISCOUNT

It is a custom among certain manufacturers and wholesale dealers to issue catalogues containing a list of prices as well as a description of the goods they are offering for sale. These catalogues are intended for the use of the trade, that is, for the retail merchants who handle the merchandise described. The prices given are called the *list*, or *catalogue prices*, and a reduction on them is usually allowed to the trade. This reduction is called **trade**, or **commercial discount,** and is quoted as a per cent. of the *list price*.

When the manufacturer or wholesale merchant wishes to change his prices, he does so either by changing the rate of discount or by giving additional discounts.

If two or more discounts are allowed, the first one is a discount off the list price, the second off the remainder, the third off what then remains, and so on.

The price remaining after all discounts are deducted is called the **net price.**

Retail merchants fix for their goods a selling price, just as the wholesale merchant has a list price for his goods. This is usually called the **marked price,** and is a per cent. of the **cost price.** Customers are sometimes given a discount off this marked price.

EXERCISE 104

1. Find the discount and net price for each of the following amounts at the rate given: $12·50 with 10% off, $15·25 with 8% off, $25·75 with 20% off, $50 with 15% off, $225 with 30% off, $626 with 25% off, $4250 with $37\frac{1}{2}$% off, $627·25 with 4% off, $982·80 with $33\frac{1}{3}$% off, $3264·80 with 3% off.

2. Find the net prices for each of the following: $625 with 20% and 10% off, $3275 with 40% and 5% off, $4000 with 25% and 20% off, $1640 with $37\frac{1}{2}$% and 10% off, $2160 with 15% and 5% off.

3. Find the net prices for: $800 with 30%, 10%, and 5% off; $680 with 20%, 20%, and 3% off; $4650 with $33\frac{1}{3}$%, 10%, and 5% off; $3400 with 10%, 10%, and 10% off; $765 with 20%, 10%, and 5% off.

4. The net price of an article listed at $16 was $12·80. What was the rate of discount? (Discount = $3·20 = $\frac{320}{1600}$ of the list price = 20%.)

5. A customer paid $3·60 for an article of which the marked price was $4. What rate of discount did he receive?

6. Goods for which the list price was $90 were sold to a retail merchant for $58·50. What was the rate of discount?

14

7. Mr. Tillson pays cash for a bill of goods and gets a discount of 5%, which amounts to $12·50. What was the amount of the bill?

8. The list price is $500. What is the difference between a discount of 40% and two discounts of 20% and 20%?

9. The catalogue price of certain goods was $1575, off which there were given two discounts of $33\frac{1}{3}$% and 6%. How much less would they have cost if there had been a single discount of 40%?

10. A florist sells a fern which is marked $6·00 at a discount of 8%. If he still makes a profit of 38%, what did the fern cost him?

11. A merchant buys from a manufacturer cloth which is catalogued at $3·20 a yard. He gets a discount of 25% and sells the cloth at a gain of 20%. At what price did he sell it?

12. What is the list price of knives for which a retail merchant pays $3 a dozen, after getting a discount of $33\frac{1}{3}$%?

13. A wholesale merchant gives 25% and 10% discount. What was the list price of goods for which he received $445·50?

14. A manufacturer can make a table for $8·40. If he is going to sell it at a profit of 20% after giving a discount of 20%, at what price must he list it?

15. A retail bookseller to whom the wholesale merchant allows discounts of 20% and 15%, buys a book of which the catalogue price is $2·50 and sells it at a profit of 30%. What does he get for it?

16. A merchant, who sells goods on 60 days' credit, allows 3% discount off all bills for cash. How much does a customer save by paying cash for a bill of $265?

17. A merchant allows a customer 5% off his bill for cash. What is the amount of a bill for which a customer pays $39·90 cash?

18. If, in the preceding Question, the merchant was marking his goods at a profit of $33\frac{1}{3}\%$, what was the cost of the goods for which he received $39·90?

19. A dealer sold a plate glass for $150 less discounts of 20%, 10%, and 5%, and made a profit of 30%. What did the plate glass cost him?

20.   Toronto, 17th. Dec., 1919.

Mr. J. D. Winch,

      Orillia.

               Bought of The Furniture Co.

Net—60 days.

Terms—3% discount if paid in 30 days.

			$	c.	$	c.
24	Kitchen Chairs (#795)	@ $ 2·50	60	00		
1	Kitchen Cupboard (#524)	@ $20·50	20	50		
2	Extension Tables (#150)	@ $18·00	36	00		
1	Hall Rack (#369)	@ $17·50	35	00		
			151	50		
	Less 40%, 10%		69	69		
					81	81

NOTE:—# means *number*. Thus #795 means No. 795 in the catalogue issued by The Furniture Co.

Is the above bill correct? What amount would settle the bill on December 29th, 1919?

21. Supply names and date, and make out bill in the above form for the following:

Bought 3 account books @ $3·50 each, 5 doz. exercise books @ 25c. each, 10 Morocco-bound note-books @ $1·25 each, 5 gross penholders @ 60c. per doz., 4 double inkstands @ $1·25 each. Discounts 20% and 5%.

TAXES

A **tax** is a sum of money levied upon the person, real estate, personal property, or income of an individual for public purposes. In every city, etc., it is usual to appoint officers, called assessors, whose duty it is to value

all taxable property, such as land, buildings, stock, income. The total of these values is called the **assessed value,** or the **assessment,** of the city, etc. The amount for which each individual is assessed is made known to him by a notice from the assessor. Try to obtain one of these notices.

The total taxes are then levied on the total assessment, and each ratepayer pays a share which is the same fraction of the total tax that his assessment is of the total assessment. The amount to be paid is calculated, usually, at the rate of a certain number of mills on each dollar of assessment. This rate is called the **rate of taxation.**

Each ratepayer receives a tax bill in some such form as this:

No. on Roll . . . .

. . . .*1919*

Mr . . . .*Con* . . . .*Lot* . . . .

TO THE MUNICIPAL CORPORATION OF NORTH MONAGHAN, Dr.

To amount of Taxes levied for 1919

On . . . . *acres as follows:*

TOTAL ASSESSED VALUE $ . . . .

1.—For County Purposes, at..5·8 Mills	$......................	....
2.—For Township Purposes, at.............................3·5 Mills	....	....
3.—For Public School Rate, Section No. 1, at........3·6 Mills	....	....
4.—For Public School Rate, Section No. 2, at........9·8 Mills	....	....
5.—For Public School Rate, Section No. 3, at........5·6 Mills	....	....
6.—For Public School Rate, Section No. 4, at........4·4 Mills	....	....
7.—For Side-walk Debenture Rate.............................	....	....
8.—For Statute Labour Commutation..........days at $1 per day	....	....
9.—For Dog Tax..	....	....
10.—For Arrears of Taxes.................	....	....
11.—For General Pub. School Rate (Co. and Tp.)....12 Mills	....	....
12.—For Separate School Rate, at ....2·2 Mills	....	....
TOTAL TAXES	....	....

RECEIVED PAYMENT

. . . .*Collector*

Exercise 105

1. What is the rate of taxation when $9 taxes are paid **on** an assessment of $4500?

Example: $4500 pays 9000 mills
                1   "   2   "

therefore rate = **2** mills on the dollar.

2. What is the rate of taxation when the taxes and the assessment are respectively: $19·80 and $3600? $178·20 and $21600? $6875 and $550,000? $141,750 and $7,875,000?

3. What amount of taxes can be raised when the assessment and the rate on the dollar are, respectively, $2500 and 6 mills? $3560 and 7 mills? $4800 and 12½ mills? $8822218 and 20 mills? $166880 and 19 mills?

4. What is the assessment when the rate on the dollar and the taxes are, respectively, 4 mills and $1640? 7½ mills and $1125? 5 mills and $13750? 16 mills and $146000? 20¼ mills and $982125?

5. A man has an income of $1900. If $700 is exempt from taxation, what amount of taxes will he be required to pay at the rate of 15 mills on the dollar? What will be his net income?

6. A municipality decides to build a bridge to cost $15000. If the assessment of the municipality is $10000000, what will a man who is assessed for $4000 have to pay toward the cost of the bridge?

7. A school section is assessed for $280000. What amount will a ratepayer who is assessed for $5200 have to contribute toward a teacher's salary of $700?

8. Mr. Henderson bought a house for $7500 on which he pays $105 taxes. If the rate of taxation be 17½ mills on the dollar, for what per cent. of the cost of the house is he assessed?

9. A city assessed for $12000000 levies a rate of 13¾ mills. What amount of taxes will the city thus obtain after paying 2½% for collection?

10. An Ontario city had an assessment on real property of \$7769128, a business assessment of \$659210, and an income assessment of \$210180. Find to the nearest cent the amount of money collected for school purposes if the total school rate was 8·9 mills on the dollar.

11. A township is assessed for \$630000. The taxes are 3½ mills for county purposes, 2⅘ mills for township purposes, 2½ mills for school purposes, 1⅖ mills for other purposes, and a poll-tax of \$1·50. If 120 persons pay the poll-tax, find the total taxes raised.

Note.—A poll-tax is a tax of a specified sum on the individual.

12. Examine the following Tax Table for a rate of 5·25 mills on the dollar and use it to find the tax paid on property assessed for \$7365.

Tax Table for Rate of 5·25 Mills

Assess-ment	Tax	Assess-ment	Tax	Assess-ment	Tax
\$1	\$0·00525	\$4	\$0·02100	\$7	\$0·03675
2	0·01050	5	0·02625	8	0·04200
3	0·01575	6	0·03150	9	0·04725

The tax on \$7000 or \$7×1000 = \$·03675×1000 = \$36·75
"   "   300 or \$3× 100 = ·01575× 100 = 1·575
"   "   60 or \$6 × 10 = ·03150× 10 = ·315
"   "   5   = ·026
         \$7365   = \$38·67.

13. Using the above Table, find the tax at 5¼ mills on \$5825. on \$690.

14. Make out tax tables like the above for rates of 24 mills, 3½ mills, and 8¼ mills.

15. Using the tax tables, find the tax on \$950 at 24 mills, \$6275 at 3½ mills, and \$9725 at 8¼ mills.

DUTIES

The preceding section explained and dealt with taxes levied for local public purposes. There are, however, other taxes levied for the support of our federal and provincial governments.

These taxes are called **duties,** and are of three classes: those levied on commodities (1) brought into the country, (2) sent out of the country, and (3) produced or manufactured in the country.

The first are called **import duties,** the second **export duties,** and the third **excise duties.**

The import and export duties, sometimes called customs duties, are payable at a Customs House; the excise duties are payable at an Inland Revenue Office.

For the purpose of duty, goods or commodities may be divided into four classes:

(1) Those which are admitted free.

(2) Those which are subject to an **ad valorem duty,** that is, a certain per cent. of their invoiced value.

(3) Those subject to a **specific duty,** that is, a certain amount a pound, a yard, a piece, etc., without reference to the value.

(4) Those subject to both an ad valorem and a specific duty.

At present goods manufactured in Great Britain and in some British colonies receive, in the matter of duty when imported into Canada, a preference over the goods imported from other countries.

A list of goods with their duties is called a **tariff.**

EXERCISE 106

1. Find the duty on the following:

(a) 250 bbl. of apples, the duty being 90c. a bbl.

(b) 625 lb. of molasses, the duty being 50c. a 100 lb.

(c) $375 worth of silk, the duty being 30%.

(d) 6 carriages invoiced at $110 each, the duty being 35%.

(e) 560 lb. of steel rivets invoiced at 6c. a lb., the duty being 75c. a 100 lb. and 25% ad valorem.

(f) 80 gal. of japan invoiced at $1·15 a gal., the duty being 20c. a gal. and $22\frac{1}{2}$% ad valorem.

2. A jeweller imports 2 doz. watches invoiced at $7·50 a watch. The duty being 30%, find the amount paid. Find also the total cost of the watches to the jeweller.

3. Knitted goods manufactured in England are subject to a duty of $22\frac{1}{2}$%, those manufactured in the United States to 35%. How much will be saved by importing $320 worth of such goods from Great Britain instead of from the United States?

4. The duty on dress goods imported from Great Britain is only 60% of that on dress goods imported from other countries. What will be saved by importing from Great Britain dress goods valued at $1600 if the regular duty imposed on such goods from other countries is 25%?

5. Find the duty on an importation of 6 tons of sugar invoiced at $3·50 a 100 lb., the duty being $31\frac{1}{2}$c. a 100 lb. What will be the total cost to the purchaser?

6. An implement dealer imports the following: 6 ploughs invoiced at $20 each; 3 binders at $125 each; 4 cultivators at $40 each; 8 wagons at $75 each.

(a) Find the amount of duty paid, if on ploughs and cultivators there is a duty of 20%, on binders $17\frac{1}{2}$%, and on wagons 25%.

(b) For how much must the whole be sold to yield a profit of $12\frac{1}{2}$% of the selling price?

7. A grocer imports 60 boxes of raisins containing 25 lb., each invoiced at 8c. a lb. He pays a specific duty of ¾c. a lb., $7·50 for freight, and $2·50 for warehouse charges. If he retails the raisins at 3 lb. for 35c., how much does he gain? What is his gain per cent.?

8. A bookseller imports a set of books invoiced at $12 and pays a duty of 25%. If he sells the books for $18, what will be his gain per cent. of the selling price?

9. An automobile invoiced at $1200 is imported into Canada, and the following duties are paid: (a) the general tariff duty of 35% on 105% of the invoice price; (b) a customs war revenue duty of $7\frac{1}{2}\%$ on 105% of the invoice price; and (c) a special war excise tax of 10% on $142\frac{1}{2}\%$ of the invoice price.

(a) Find the total duty.

(b) Find also at what price the automobile must be sold to make a profit of 20%.

(c) What difference would there be in this price if there were no duty?

INSURANCE

Every year much property is destroyed or damaged by fire. So seriously do most persons fear the loss which they might suffer from this cause that it is now an almost universal custom for property owners to enter into an agreement with certain companies by which, in return for a stated annual payment, the companies will pay a specified sum for any loss sustained.

These companies are called Fire Insurance Companies. The agreement to compensate for loss is called **insurance.**

The written contract between the property owner and the company is called a **policy.**

The sum specified to be paid in case of loss is called the **amount,** or **face,** of the policy, or the **risk.**

The cost of insurance, that is, the payment made by the property owner to the company, is called the **premium,** and it is a per cent. of the risk.

Rates for insurance vary for different locations, for different businesses, for different kinds of property. Why?

EXERCISE 107

1. What is the premium on a $500 policy at $\frac{3}{4}\%$?

2. What is the premium on a $2200 policy at 65c. for $100?

3. An hotel worth $12500 is insured for $\frac{4}{5}$ of its value at $1\frac{4}{5}\%$. What is the premium?

4. Find the face of the policy on which $19·20 is paid as a premium at $\frac{3}{4}\%$?

5. What amount of risk can I get for $25·60 at 80c. per $100?

6. A dealer insured his stock of goods for $64000, paying as premium $96. Find the rate of insurance.

NOTE.—What fraction is the premium of the amount of the policy? What per cent.?

7. A manufacturer insured his factory valued at $12000 for $87\frac{1}{2}\%$ of its value and paid a premium of $280. What was the rate of insurance?

8. A building worth $6000 is insured for $4500 at 9%. In case the building is destroyed, what will be the owner's net loss?

9. A factory worth $8000 is insured for $5000 at $2\frac{1}{2}\%$, and is damaged by fire to the extent of $6000. What is the owner's net loss?

10. A farmer insures his house for $1200 and its contents for $750, his barn for $1400 and its contents for $800. Find the total premium paid at $\frac{4}{5}\%$.

11. If brick houses are insured at 75c. a $100, while frame houses are charged 1%, what is the difference

between the premium paid for insuring a brick house and a frame house for $1840 each?

12. Mr. Leslie insured some property at $\frac{9}{10}$%, and found that in case of loss he would recover the value of the property as well as the premium paid. If the premium was $45, what was the value of the property?

13. For what sum must grain valued at $2485 be insured at $\frac{3}{5}$%, so that in case of loss the owner may recover the value of the grain together with the premium paid?

14. Mr. Hay insures at $1\frac{1}{4}$% his goods invoiced at $23800 for 80% of their value. If they should be destroyed by fire, what would be Mr. Hay's total loss?

15. A stock of goods is insured in the London Assurance Co. for $5500, in the Queen Assurance Co. for $3500, and in the Mutual Assurance Co. for $6500. If the goods are damaged to the extent of $9000, how much of the loss will each Company have to pay?

16. An insurance company which charged a rate of $\frac{4}{5}$%, received $60 for insuring a house for $\frac{3}{4}$% of its value. Find the value of the house.

17. Samuel Long insured property as follows: House valued at $12,000 for $\frac{2}{3}$ of its value at $1\frac{1}{2}$%; plate glass windows for $750 at $1\frac{3}{5}$%; furniture for $6000 at $1·20 for $100 for 3 years; garage for $1500 at $1·40 for $100 for 3 years; and an automobile for $1750 at 2%. What annual rate is he paying for his insurance?

SIMPLE INTEREST

Just as one who lives in a house or on a farm which is not his own has to pay for the use of that house or farm, so a person who borrows money has usually to pay for the use of that money.

Money paid for the use of borrowed money is called **interest**. The money borrowed is called the **principal**.

The total sum paid to the person or persons from whom the money is borrowed is called the **amount.**   Of what two sums is the amount made up?

The interest for one year is a certain per cent. of the principal.   This per cent. is called the **rate of interest per annum.**

When one buys goods on credit he is, in reality, borrowing money, and is, therefore, sometimes charged interest on the value of the goods bought.   So, too, a banker with whom money is deposited uses borrowed money and usually pays interest on it.

It is the custom for the borrower to give to the person from whom he borrows a written acknowledgment of his debt as well as a promise to repay the money.   This written document is called a **promissory note,** the form of which is as follows:

$250·00

Ottawa, November 1st, 1919.

Three months after date I promise to pay James H. Broadbent, or order, Two Hundred and Fifty . .00/100 Dollars for value received, with interest at 5% per annum.

J. D. CRAWFORD.

The person who signs the note (J. D. Crawford) is called the **maker.**

The person to whom the money is to be paid (James H. Broadbent) is called the **payee.**

The sum borrowed ($250) is called the **face of the note.**

When dates are given, interest for a part of a year is usually calculated by days.   For instance, a $250 account due on November 1st. but not paid until February 1st.

would be 92 days overdue, and interest on it for that period at 5% would, therefore, be $\frac{92}{365}$ of the interest for one year, or $\frac{92}{365}$ of $\frac{5}{100}$ of $250.

EXERCISE 108

1. Find the interest on $640 for 3 years at $5\frac{1}{2}\%$ per annum. [Interest = ($\frac{11}{200}$ of $640) $\times$ 3.]

2. Find the interest on $200 for 1 year at 6% per annum; $350 for 2 years at 4% per annum; $50 for $2\frac{1}{2}$ years at 3% per annum; $280 for $1\frac{1}{4}$ years at $3\frac{1}{2}\%$ per annum; $75·50 for $6\frac{2}{3}$ years at $4\frac{1}{2}\%$ per annum.

3. Find, to the nearest cent, the interest on, and the amount of, $428·25 for 2 years and 3 months at 6%. (Interest = $\frac{27}{200}$ of principal. Therefore amount = $\frac{227}{200}$ of principal.)

4. Find, to the nearest cent, the interest on, and the amount of, $562·85 for 3 years and 73 days at 5%; $4200 from May 5th, 1917, to September 8th, 1917, at $4\frac{1}{2}\%$; $3725 from June 10th, 1917, to May 8th, 1918, at $3\frac{1}{2}\%$; $6250 from January 8th, 1918, to March 6th, 1919, at 4%; $10000 from April 30th to October 8th at $5\frac{1}{2}\%$.

5. On what principal is the interest $45 in $2\frac{1}{2}$ years at 6%? ($\frac{15}{100}$ of principal = interest.)

6. On what principal is the interest $60 in 2 years at 6%? $144 in $1\frac{1}{3}$ years at $4\frac{1}{2}\%$? $25·50 in 6 months at $4\frac{1}{4}\%$? $198 in 90 days at $5\frac{1}{2}\%$?

7. In what time will $250 give $75 interest at 6%? (Interest for 1 year = $\frac{6}{100}$ of $250 = $15.)

8. In what time will $625 give $12·50 at 4%? $4280 give $748 interest at 5%?

9. Find the rate per cent. per annum when the interest on $840 for $2\frac{1}{2}$ years is $105. (Interest for 1 year = $42 = $\frac{42}{840}$ of principal = 5% of principal.)

10. Find the rate per cent. per annum when the interest on $6000 for 1 year is $240; on $1600 for 9 months is $72; on $2190 for 80 days is $16·80.

11. At what rate per cent. per annum will $1000 amount to $1030 in 8 months? $620 to $713 in 3 years? $360 to $403·20 in 4 years?

12. On January 5th, 1918, Mr. Jackson borrowed $250 and agreed to pay interest thereon at $4\frac{1}{2}\%$ per annum. How much did he owe on April 5th, 1918?

13. On June 6th, 1918, Thomas Black of Sarnia bought from Henry Stone goods valued at $125 and agreed to pay for them in four months with interest at 6% per annum. On what day was the account due? How much did Mr. Black owe Mr. Stone on that day?

14. Write a receipt for the interest on a loan of $80 for 5 months with interest at $5\frac{1}{2}\%$ per annum.

15. If you deposit in a Savings Bank $120 on January 1st, and $180 on February 3rd, how much should there be to your credit on March 31st. if the bank allows you interest at the rate of $3\frac{1}{2}\%$ per annum?

16. A Savings Bank which pays interest at the rate of 4% per annum makes up its interest on June 30th. If the interest on any account is equal to the difference between the interest up to June 30th. on all sums withdrawn and the interest on all sums deposited, find the amount of Henry Rust's account on June 30th, if on January 10th. he deposits $200, on February 15th. withdraws $150, on February 25th. deposits $250, on April 4th. deposits $400, on May 8th. withdraws $200, on June 30th. deposits his interest.

17. On January 6th. a customer buys goods valued at $136, but does not pay his bill until June 4th. How much should he then pay if the merchant charges 4% interest on all accounts?

18. On September 4th. I buy goods worth $200, and am allowed 30 days in which to make payment. After that time I agree to pay 3% interest from date of purchase. How much shall I owe on December 10th.?

19. Bankers allow interest for any month on the minimum balance for that month. Using this balance,

find the month's interest at 3% per annum and also at 3½% per annum on each of the following bank accounts:

(*a*) Balance Feb. 1st. $465·75; Feb. 3rd. deposited $72; Feb. 10th. withdrew $264·50; Feb. 17th. deposited $110; Feb. 21st. withdrew $69.

(*b*) Balance May 1st. $625; May 11th. deposited $435; May 17th. withdrew $208; May 26th. withdrew $150; May 30th. deposited $400.

(*c*) Balance Sept. 1st. $1250; Sept. 6th. withdrew $375; Sept. 16th. deposited $415; Sept. 24th. deposited $590; Sept. 29th. withdrew $1536.

BANK DISCOUNT

A great deal of a bank's business consists in discounting notes, that is, in cashing or paying them before they are due.

If a person, say, Mr. Sidney Johnston, wishes to borrow money from a bank, he may do so on one or more of several kinds of notes, but the form of the one commonly used is as follows:

$1000·00

Chatham, Dec. 10th, 1918.

Ninety days after date I promise to pay to the order of Timothy Newton One Thousand.......00/100 Dollars at the Standard Bank, Chatham.

Value received.

SIDNEY JOHNSTON.

Mr. Newton endorses this note by writing his name across the back of it, and so becomes responsible for its payment by Mr. Johnston or by himself.

Mr. Johnston, or Mr. Newton for him, may then take the note to the Standard Bank and receive cash for it,

provided the bank officials are satisfied that both men are reliable.

The bank, however, will not pay the full amount of $1000, but will deduct a percentage of it; and the sum so deducted is called **bank discount.**

If the note is presented at the bank on December 10th, 1918, and the bank's rate of discount is 7% per annum, the discount will be $\frac{93}{365}$ of $\frac{7}{100}$ of $1000, or $17·84.

The amount which the bank would pay on the note would be $1000 − $17·84, or $982·16, which is called the **proceeds** of the note.

Although the time named in the note, that is, the *nominal* time, for payment is 90 days, the discount for 93 days is taken, because in Canada a note is not **legally** due until *three days* after it is **nominally** due.

These three extra days are called **days of grace,** and banks always include them in the term of discount.

If the note is not presented at the bank until, say, January 1st, 1919, the bank will then deduct only 73, that is, 93–20, days' discount.

The number of days from the day on which the bank gets a note to the day on which the note is legally due, is called the **unexpired time** or the **term of discount.**

Since the above note was not worth $1000 until it was due, this sum is called **the value of the note at maturity.** In interest-bearing notes the value at maturity includes the interest due on the legal date for payment.

The bank discount on a note is *a percentage of its value at maturity for the term of discount at the bank rate of discount.* The proceeds are the value at maturity less the bank discount.

EXERCISE 109

1. On January 4th, 1919, William Lynch gives to the Bank of Montreal his note, endorsed by John Lawrence, for $300 payable in 3 months without interest. Write the note, dating it January 4th, 1919. On what day is it legally due? Find the proceeds if the bank's rate of discount is 6%. (Bank discount = $\frac{93}{365}$ of $\frac{6}{100}$ of $300. Proceeds = $300 − bank discount.)

2. Find the date of maturity, the term of discount, the discount, and the proceeds of the following:

(a) A 60-day note dated October 4th, 1918, for $400 without interest, and discounted on the same day at 5% per annum.

(b) A 90-day note dated May 3rd, 1919, for $1000 without interest, and discounted immediately at $5\frac{1}{2}$%.

(c) A 4-month note dated June 7th, 1919, for $6000 without interest, and discounted July 3rd, 1919, at 5%.

(d) A 90-day note dated July 6th, 1918, for $6000 without interest, and discounted July 26th, 1918, at $5\frac{1}{2}$%.

(e) A 6-month note dated June 16th, 1919, for $1500 without interest, and discounted July 26th, 1919, at 6%.

3. If a 60-day note without interest is immediately discounted at 7%, what fraction of its value at maturity would the discount be? What fraction of its value at maturity would the proceeds be? If the proceeds were $3605·90, what would be the value of the note at maturity?

4. For what sum must a 90-day note be drawn, so that when it is discounted immediately at 5% the proceeds may be $1441·40?

5. For what sum must a 3-month note dated May 10th, 1919, be drawn, so that if it is discounted on June 2nd, 1919, at $6\frac{1}{4}$%, the proceeds may be $3605?

6. On February 6th, 1920, R. T. Sutton borrowed $600 from M. L. McBain on a 6-month note bearing interest at 5% per annum. Write the note. On what day was it legally due? What was its value on the day of maturity?

15

7. If Mr. McBain endorsed the note in the preceding Example and sold it to the Royal Bank on May 26th, 1920, what did he get for it if the bank's rate of discount was $5\frac{1}{2}\%$?

8. Find the day of maturity, the value at maturity, the term of discount, the discount, and the proceeds of the following:

(*a*) A 90-day note dated May 17th, 1919, for $2500 with interest at 5%, and discounted at the bank on June 3rd, 1919, at 7%.

(*b*) A 4-month note dated August 14th, 1920, for $1200 with interest at 5%, and discounted at the bank the same day at 5%.

EXERCISE 110 (REVIEW)

1. A man buys two farms. For the one, containing 65 acres, he paid $48 an acre, and for the other, containing 85 acres, he paid $64 an acre. If he sells the first at a gain of $12\frac{1}{2}\%$ and the second at a loss of $6\frac{1}{4}\%$, will he gain or lose on the whole, and how much?

2. A collector receives on a certain day payments for the following: $360 and interest for 3 mo. at 6%; $150 and interest for 1 yr. at 5%; $800 and interest for 4 mo. at $4\frac{1}{2}\%$; $250 and interest (simple) for $1\frac{1}{2}$ yr. at 4%. Write a receipt for the full amount paid in each case.

3. In a certain season a ball team won 54 games and lost 16. Find its winning average, that is, the per cent. of games won.

4. A gentleman bought a $1200 automobile at 15% discount, and, after using it for two seasons, sold it for $33\frac{1}{3}\%$ less than he paid for it. How much did he receive for it?

5. Three persons are partners in a business in which the first invests $1200, the second $1600, and the third $800. What per cent. of the business does each own? How should a profit of $2400 be divided among them?

6. Mr. Saunders insures a stock of goods worth $14,000 for 80% of its value at $1\frac{1}{8}$%. If the goods are entirely destroyed by fire, what is the loss including the premium paid?

7. A manufacturer sells an article for $3·24. The material cost him $1·55, the labour 55c., and other expenses 40c. Find his gain per cent.

8. For selling some property on a commission of $2\frac{1}{2}$%, an agent received $29·50. For what amount did he sell the property? What did his employer receive for it?

9. A dealer bought 600 tons of coal at $8 a ton. He sold $33\frac{1}{3}$% of it at an advance of 25%. He then sold 20% of the remainder at an advance of 20%, and the rest he sold at $9 a ton. Find his average gain per ton.

10. An agent receives $1,624 to invest after deducting a commission of $1\frac{1}{2}$% on the amount invested. Find his commission.

11. The sum of the principal and the interest for 4 mo. at 6% per annum was $326·91. Find the principal.

12. At $\frac{1}{2}$c. a lb. and 35% ad valorem, what will be the duty on 280 lb. of candied orange-peel invoiced at 15c. a lb.?

13. Make out a 90-day note for $600, dated to-day and payable to Henry Martin at your nearest bank. Discount it immediately at 6% and find the proceeds.

14. A roll of 120 yd. of cloth was sold for $216 at a loss of 10%. For what should it have been sold a yard to give a profit of 20%?

15. On a demand note for $1000, dated Jan. 15th, 1919, and drawing interest at 6%, the following payments were made: Feb. 26th, 1919, $125; March 28th, 1919, $220; June 17th, 1919, $400. How much was due on Sept. 6th, 1919?

16. A premium of $66 is paid for insuring a cargo of grain for $\frac{3}{4}$ of its value. If the rate of insurance is $1\frac{1}{4}$% and the grain worth 80c. a bushel, find the number of bushels in the cargo.

17. How much tax must be paid by a man whose property is assessed for $8500 if the rate is $6\frac{1}{2}$ mills?

18. Make out a bill for the following:

Bought of the Red Book Co.: 4 gross lead-pencils @ $3·25; 6 doz. qt. ink @ $7·20; 40 reams foolscap @ $1·20; 10 gross pen points @ 35c.; 120 lb. blotting-paper @ $7·50 a 100 lb. Discounts 10%, 5%.

19. A note for $300 was dated October 1st, 1919, and drawn for 3 mo. If it was discounted at the bank on November 10th, 1919, at 6%, find the proceeds.

20. A merchant buys a book, the list price of which is $7·50, at a discount of 30%, and sells it for $8. What is his gain per cent.?

21. A grocer buys 45 doz. eggs at 20c. a doz., but finds that 15 eggs are broken. At what price a dozen must he sell the remainder to clear $16\frac{2}{3}\%$ on the total cost?

22. In selling a hat which was marked at $36, a milliner found that it was marked 60% above cost. How much can she reduce the price and still make a profit of 40%?

23. How many pounds of cream of which 30% is butter fat can be taken from 600 lb. of milk of which 4·5% is butter fat?

24. The protein in food forms muscle, while the carbohydrates and fats form fat. What per cent. is the muscle product of the fat product of each of the following: Eggs which contain 12·7% protein and 8·8% fat? Beef which contains 15·6% protein and 16·6% carbohydrates? Potatoes which contain 1·5% protein, 0·1% fat, and 14% carbohydrates? Wheat which contains 7·8% protein, 1·2% fat, and 52% carbohydrates?

25. Mr. Stinson wants to rent a house which cost him $6000 at such a rate as will yield him a clear income of 6% a year on his investment. He pays a tax of 18 mills on the dollar on $\frac{3}{4}$ the value of the house and insurance on $\frac{1}{2}$ its value at $\frac{3}{5}\%$. What monthly rate must he ask?

26. Goods were bought for $1200 subject to discounts of 25% and 16⅔%. They were sold at an advance of 33⅓% on actual cost. Find the gain if 10% of the sales were bad debts.

27. The food which plants obtain from the soil consists chiefly of nitrogen, phosphorus, and potash. If wheat contains 2·4% of nitrogen, ·9% of phosphorus, and ·6% of potash, how many lb. of each of these elements will be taken from the soil of a ten-acre field of wheat yielding 30 bu. to the acre?

28. A dressmaker imported 75 yd. of lace invoiced at $120. She paid 35% duty and sold the lace at a gain of 37½%. What was the selling price of one yard?

29. Find to the nearest cent the interest on $720·50 from June 4th, 1919, to Nov. 7th, 1919, at 4% per annum.

30. An agent sold for a cloth manufacturer 1020 yd. of cloth at $2·00 a yard, on a commission of 2%. He invested the net proceeds in wool at 40c. a pound, on a commission of 2%. How many pounds of wool did he buy?

MEASUREMENTS

EXERCISE 111

1. A rectangular field is 5 chains 17 links long and 3 chains 8 links wide. Find its perimeter in feet.

2. The perimeter of a rectangular plot is 7½ rods. The length is 1½ times the width. Find the dimensions in feet.

3. A field 120 rods long and 80 rods wide is inclosed by a wire fence 7 strands high, with posts every 15 ft. Find the cost of the posts at 25c. each and of the wire if one strand costs $1·75 for 40 rods.

4. In a field 440 yards long and 200 yards wide tile drains are laid the full width of the field, 4 rods apart, the two outer drains being each 4 rods from the edge of the field. What will the tiles cost at $35 a hundred, each tile being 1 foot long?

5. Find the area of a rectangle 36 ft. long and 216 in. wide.

6. How many acres in a farm 400 rods long and 320 rods wide?

7. Find the area of the whole surface of a rectangular block 7½ ft. long, 4 ft. wide, and 18 in. high.

8. What will it cost to paint the external surface of a closed box 8 ft. long, 4 ft. wide, and 2 ft. 9 in. high, at 15c. a sq. ft.?

9. Find the cost of plastering the ceiling and walls of a room 20 ft. long, 18 ft. 3 in. wide, and 10 ft. 8 in. high, at 62½c. a sq. yd., making no allowance for openings.

10. If a bundle of 100 laths will cover 5 sq. yd. of surface, how many bundles will be required for the walls and ceiling of a room 15 ft. long, 12 ft. wide, and 10 ft. high, allowance being made for a door 7 ft. by 3 ft. and a window 8 ft. by 3 ft.?

11. How many yards of carpet 27 in. wide will it take to cover the floor of a room 20 ft. 8 in. long and 13 ft. 6 in. wide?

12. It is estimated that 1000 shingles will cover 100 sq. ft. of surface. At \$4·50 a thousand what will be the cost of the shingles for a barn roof each side of which is 30 ft. wide and 60 ft. long?

13. The perimeter of a square field is 64 rods. How many acres does it contain?

14. A lot 176 ft. deep and 66 ft. frontage was sold at \$40 a foot frontage. How much was this an acre?

15. At the rate of 3 miles an hour, how long will it take to walk 8 times round the outside of a rectangular plot which contains 3½ acres and the length of which is 35 rods?

16. What will be the perimeter of a rectangular plot the width of which is 18 yd. 2 ft. 3 in., and the area of which is equal to that of a square the perimeter of which is 120 yd.?

17. For the floor of a living-room 30 ft. long and 24 ft. 6 in. wide a lady purchased a rug 26 ft. 4 in. long, and found that it left a uniform margin of the room uncovered. What was the width of the rug?

18. How many cubic yards of earth are removed in excavating for a cellar 36 ft. long, 24 ft. wide, and $4\frac{1}{2}$ ft. deep?

19. How many cubic feet of concrete are there in the walls of a root cellar the inside dimensions of which are 14 ft., 10 ft., and $7\frac{1}{2}$ft., the walls being 8 in. thick?

20. How high is a rectangular solid containing 693 cu. ft. if the dimensions of its base are 9 ft. and 7 ft.?

21. A farmer places in the attic of his house a rectangular water-tank having a capacity of 800 gal., and the end of which is 4 ft. square. What is the length of the tank if a cubic foot holds $6\frac{1}{4}$ gal.?

22. The weight of a cubic foot of iron is $7 \cdot 53$ times that of a cubic foot of water. If a cubic foot of water weighs 1000 oz., what is the weight of a bar of iron 9 ft. long, 4 in. wide, and 6 in. thick?

23. Water flows into a rectangular tank at the rate of 15 gal. a minute. How high will the water rise in the tank in one hour if its dimensions are 9 ft. by 18 in. and a gal. contains $277 \cdot 27$ cubic inches?

24. A bushel contains $2218 \cdot 2$ cubic inches. How many bu. of oats are there in a bin 11 ft. 6 in. long, 6 ft. wide, and 5 ft. deep?

25. A carpenter constructs a coal-bin $10\frac{1}{2}$ ft. long, 8 ft. wide, and $7\frac{1}{2}$ ft. deep. How many tons will it hold if it takes 35 cubic feet for one ton?

26. A cow requires 3953 cu. ft. of fresh air an hour. If a ventilating flue, the opening of which is 6 in. square, admits air at the rate of 250 feet a minute, how many of these flues would be required to supply a stable with the air necessary for 24 cows?

27. How many paving bricks $7\frac{1}{2}$ in. long and 4 in. wide will it take to pave 2 miles of street 45 ft. wide?

28. A rectangular garden 64 ft. long and 35 ft. wide is divided into 4 equal rectangular plots by two paths, each 2 yd. wide, running through its centre, one from end to end, the other from side to side.   How many square yards are there in the area of the paths?

29. How many blocks of ice 3 ft. long, 2 ft. wide, and 2 ft. thick, will it take to fill an ice-house 40 ft. long, 28 ft. wide, and 16 ft. high, if there are 2 ft. of sawdust between the ice and the floor, walls, and ceiling?

30. A dealer has in his yard a pile of wood 42 ft. long, 32 ft. wide, and 10 ft. high.   What is this wood worth at $12 a cord?

31. (*a*) How many board feet are there in:

120 boards each 12 ft. 6 in. $\times$ 8 in. $\times$ 1 in.
65 planks each 16 ft. $\times$ 10 in. $\times$ 2$\frac{1}{2}$ in.
48 scantlings, each 16 ft. $\times$ 2 in. $\times$ 4 in.?

(*b*) How much will this lumber cost at $45 a thousand?

32. How much 2-in. lumber will it take to make a box without a cover, the outside dimensions of which are: 5 ft. 4 in., 2 ft. 8 in., and 1 ft. 8 in.?

33. Find the sum of the following expenses for opening a new street 4 rods wide and 60 rods long:

Constructing sewer, @ $1·25 a linear ft.
Levelling and grading whole street @ $1·50 a sq. rd.
Paving—36 ft. wide, @ $1·20 a sq. yd.
Two side-walks—each 6 ft. wide, @ 15c. a sq. yd.
Trees on each side 30 ft. apart, @ 75c. each.

EXERCISE 112 (GENERAL REVIEW)

1. (*a*) In 1919 the number of automobiles in Canada was as follows: Ontario 139,288; Saskatchewan 56,402; Quebec 31,777; Alberta 34,362; Manitoba 33,896; British Columbia 19,161; Nova Scotia 10,030; New Brunswick 8,142; and Prince Edward Island 1,132.   Find the total.

(b) Taking the population of Canada as 8,835,000, what is the average number of persons for each automobile?

2. On June 15th, 1919, Captain Alcock and Lieutenant Brown crossed the Atlantic from Newfoundland to Clifden, Ireland, in an aeroplane, in 16 hr. 12 min., at an average rate of 120 miles an hour. Find the distance.

3. During the navigation season of 1919 on the Great Lakes, it is said that for 216 days an average of one vessel passed Windsor every 20 min. 45 sec. Find the number of vessels.

4. In canning raspberries or blackberries, 1 pt. of sugar is required for 2 qt. of berries, and for currants, 1 qt. of sugar is required for 3 qt. of currants. At $7·50 a cwt. find the cost of the sugar required to can one crate (24 qt.) of each of these fruits if a pt. of sugar weighs one lb.

5. (a) If a clean field produces 62 bu. of corn to the acre while a weedy one produces only 35 bu., what is the loss caused by weeds on a 10-acre field if corn is worth $1·10 a bu.?

(b) For how many days would this loss pay for a hired man at $2·25 a day?

6. In 25 gal. of a mixture for spraying, there are 5 lb. of unslaked lime worth 5c. a lb., and 3 lb. of copper sulphate worth 40c. a pound. Of this mixture 400 gal. will spray 2 ac. of potatoes, and the yield of potatoes is thereby increased from 87 bu. an ac. to 174 bu. an ac. If potatoes are worth $1·60 a bu., what is the net gain per acre due to spraying?

7. (a) Divide 143689 by 231, using three factors.

(b) Find the H.C.F. and the L.C.M. of 1656 and 3087.

8. (a) Find the sum of:

$(\frac{1}{2} + \frac{1}{3})$, $(\frac{2}{5} - \frac{1}{4})$, $(\frac{1}{2} + \frac{1}{3} \times \frac{1}{4})$ and $(\frac{2}{3} - \frac{1}{6} \div \frac{2}{5})$.

(b) Simplify:

$(4\frac{2}{5} + 2\frac{1}{3} + 3\frac{1}{12} + 1\frac{5}{6}) \div (4\frac{5}{18} - 3\frac{5}{9})$.

9. (a) Simplify:

$6\frac{1}{7} \times 3\frac{1}{2} - 2\frac{1}{3} \div \frac{5}{7} \times 1\frac{3}{7} + 1\frac{3}{8}$.

(b) Simplify:

$(6 \cdot 842 + 3 \cdot 158) \div \cdot 125$.

10.(a) Reduce to 4 places of decimals:

$\frac{3}{16}, \frac{17}{125}, \frac{2}{13}, \frac{4}{23}, \frac{5}{31}$.

(b) Express 82 sq. yd. 4 sq. ft. as a fraction and then as a decimal of 97 sq. yd. 2 sq. ft.

11. Which has the faster rate—a train which runs 120 yd. in 5 seconds, or one which runs a mile in 75 seconds?

12. Two men are to divide $54 between them in such a manner that one is to get $1½ as often as the other gets $1¾. How is the money divided?

13. A piece of goods is shortened by tucking. It was originally ¾ yd. long, but after tucking it was only ½ yd. long. How many ⅛-in. tucks were put in it?

14. Make out in proper form, supplying names and date, a bill for the following:

22½ lb. sugar @ $9·60 a cwt.; 32 lb. tea @ 65c.; 12 lb. coffee @ 42⅜c.; 15 lb. biscuit @ 3 lb. for 50c.; 20 lb. cheese @ 27½c.; 5 pkg. oatmeal @ 35c.; 2 doz. grape fruit @ 4 for 25c.; 6½ lb. bacon @ 62½c.; 5 gal. coal-oil @ 32½c.; 2 bbl. salt @ $1¾.

15. Make out the account of Morris Bros., Hardware Merchants, against J. D. Gay, from the following items:

Nov. 1—Account rendered $50·35; Nov. 5—Nails $5·60; Nov. 6—Tin roofing $22·40; Nov. 10—Shovels $2·50; Nov. 12—Mr. Gay paid on account $65·00; Nov. 17—Cement $14·00; Nov. 24—Mr. Gay gave cheque on account $25·00; Nov. 28—Glass $15·00.

16. At a school examination the paper in history was valued at 150 marks. On this paper 4 pupils obtained, respectively, the following marks: 144, 135, 125, and 120. What per cent. of the total marks did each pupil take, and what was the average per cent. for the four pupils?

17. Mr. Jones bought a city lot having a frontage of 340 ft. at $45 a foot. He borrowed the money at 5% per annum and, at the end of one year, sold the lot at a gain of 20% on the original cost. Find his net gain.

18. Mr. Byer bought for 60c. on the dollar a bankrupt stock valued at $10,000. He sold $\frac{1}{4}$ of it at 20% advance on what it cost him, $\frac{2}{5}$ of it at an advance of 25%, and the remainder at an advance of $33\frac{1}{3}$%. If 5% of the sales were not collectible, find his total net gain, allowing $400 for expenses.

19. A tax of $4000 is levied on a school section of which the assessment is $250,000. Find the rate. Find also the tax paid by a ratepayer having an income of $1900, of which $700 is exempt from taxation.

20. A man borrowed $320 on May 16th, 1919. What sum would cancel the debt on Oct. 9th, 1919, if interest was at the rate of $5\frac{1}{2}$% per annum?

21. An agent collected for his employer 80% of debts amounting to $2000. How much did he remit to his employer after deducting a $4\frac{1}{2}$% commission?

22. A dealer in real estate buys a house for $2400 and rents it expecting to make $6\frac{1}{2}$% interest on his investment. His annual expenses are: Taxes on $\frac{5}{8}$ of the value at 16 mills; insurance on $\frac{2}{3}$ of the value at $\frac{2}{5}$%, and $21·60 for repairs. What monthly rent must he charge?

23. A merchant imports goods invoiced at $2·00 a yd., and pays thereon an ad valorem duty of $12\frac{1}{2}$% and a specific duty of 5c. a yd. At what price a yd. must it be marked so that a discount of 8% will still leave a profit of 20%?

24. At simple interest a sum of money amounts in 2 yr. to $330 and in 4 yr. to $360. Find the principal and the rate.

25. A man borrowed $600 at 5% interest and paid back $150 at the end of each year, to pay the interest and reduce the debt. How much will he owe at the end of the third year?

26. A 70-day note for $250 is dated July 1st, 1919, and bears interest at 6% per annum. If it is sold to the bank at 7% discount on Aug. 4th, 1919, what will the proceeds be?

27. What will it cost to fence a school ground 16 rd. long and 10 rd. wide with woven wire costing 27c. a rod, posts 11 ft. apart costing 30c. each, and a top scantling $2\frac{1}{2}$ in. by 4 in. costing $32 a thousand?

28. How much 1-in. lumber will it take to make a closed box with butt joints if the outside dimensions are: 5 ft. 2 in., 4 ft. 8 in., and 3 ft. 6 in.?

29. A cubic foot of soil is dug up and is found to weigh 120 lb. It is then dried and now weighs only 104 lb. How many tons of water would there be in an acre of this soil 1 ft. deep?

30. What depth of rain would have to fall on an acre of ground to give moisture equivalent to that in Question 29, if a cubic foot of water weighs 1000 oz.?

SECTION VI

EXERCISE 113 (GENERAL REVIEW)

1. Find the prime factors of 7392 and of 24720.

2. Find the H.C.F. of:
(a) 5325 and 8307.
(b) 5210 and 5718.
(c) 805, 1311, and 1978.

3. Find the L.C.M. of:
(a) 12, 14, 20, 24, 54, and **63**.
(b) 144, 186, and 496.
(c) 255, 239, 1023.

4. Find the value of:

(a) $\dfrac{84 \times 63 \times 75 \times 105}{81 \times 35 \times 50 \times 28}$.

(b) $\dfrac{64 \times 57 \times 48 \times 54}{36 \times 76 \times 72}$.

5. Simplify:

(a) $5\frac{1}{5} + 2\frac{5}{8} - 3\frac{3}{10} + 6\frac{1}{2} - 10\frac{1}{4}$.

(b) $(3\frac{1}{8} \times 4\frac{1}{6} - 5\frac{1}{7} \div 3\frac{3}{5}) \div (4\frac{4}{11} - 3\frac{4}{7})$.

(c) $\dfrac{3\frac{3}{4}}{4\frac{2}{7}} - \dfrac{3\frac{2}{5}}{4\frac{1}{4}} + \dfrac{\frac{4}{7}}{2\frac{1}{2}}$.

(d) $\dfrac{4\frac{1}{7} \times 1\frac{13}{15} \times 3\frac{9}{11}}{5\frac{5}{11} - 4\frac{2}{5}}$.

6. (a) Divide $2\frac{4}{5} + \frac{3}{7}$ of $8\frac{3}{4}$ by $2\frac{3}{4} \times 2\frac{8}{9} + \frac{3}{10}$.

(b) By how much does $\frac{5}{8}$ of $\frac{4}{10} - \frac{7}{9}$ of $\frac{4}{21}$
exceed $\frac{5}{6}$ of $\frac{2}{15} - \frac{3}{8}$ of $\frac{4}{15}$?

7. Find the value of:

(a) $4 \cdot 37 \times 83 \cdot 62 \times \cdot 074$.

(b) $\dfrac{7 \cdot 35 \times \cdot 0143}{15 \cdot 015} - \dfrac{\cdot 152 \times \cdot 033}{2 \cdot 09}$.

(c) $\dfrac{21 \cdot 7 \times \cdot 087}{2 \cdot 03} + \dfrac{101 \cdot 01 \times \cdot 319}{2 \cdot 639}$.

8. (a) Express 5 ft. $9\frac{3}{4}$ in. as a fraction of 3 ft. $2\frac{1}{2}$ in.

(b) If a man 5 ft $9\frac{3}{4}$ in. in height casts a shadow of 3 ft. $2\frac{1}{2}$ in., what is the height of a steeple which at the same time casts a shadow of 88 ft. $6\frac{1}{2}$ in.?

9. A man experimented with seven hogs to ascertain how many pounds of corn it would require to produce 100 pounds of pork. The result for the different hogs was as follows:

586 lb., 540 lb., 587 lb., 579 lb., 530 lb., 500 lb., and 479 lb.

(a) Find the average number of pounds required.

(b) At this average, what should be the price of pork a hundredweight if corn is selling at $1·28 a bushel?

10. A cow requires annually 3 tons of hay worth $18 a ton; 1000 lb. of chopped grain worth $38 a ton; and pasture worth $15. If a Jersey cow gives 25 lb. of milk daily for 300 days in the year and a "scrub" cow gives only 15 lb., find the yearly profit from each of these cows if milk is worth 5c. a lb.

11. Find the amount of the following:

3258 lb. of wheat @ $2·10 a bu.; 1768 lb. of oats @ $1·02 a bu.; 1836 lb. of barley @ $1·56 a bu.; 729 lb. of peas @ $2·90 a bu.; 2820 lb. of corn @ $1·82 a bu.; $4\frac{1}{2}$ bu. of clover seed @ 55c. a lb.

12. A man and team can till thoroughly 3 ac. a day or carelessly 5 ac. a day. If careless tilling produces 38 bu. of oats an acre and thorough tilling 56 bu., what is the loss due to careless work on a 30-acre oat-field, when oats

are worth 90c. a bushel and a man and team are worth
$6·50 a day?

13. The statement of the Spring Brook Cheese and
Butter Company for the year 1919 gives the following
information:

Lb. of milk received................ 1,547,524
Lb. of cheese manufactured........ • 128,278
Lb. of milk for 1 lb. cheese.......... 11·36
Lb. of butter manufactured........ • 4,322·75

Average price for 1 lb. cheese....... • 27·085c.
Average price for 1 lb. butter....... • 45·10c.
Other receipts..................... $63·51
Cost of manufacturing 1 lb. cheese·.. 2c.
Cost of manufacturing 1 lb. butter.. • 10c.
Other expenses................... $5019·40

Calculate: (a) Amount of milk used for cheese; (b)
Amount of milk used for butter; (c) Amount of milk to
make 1 lb. butter; (d) Total receipts; (e) Total expendi-
ture; (f) Balance for profits; (g) Amount of profit for 1 lb.
of milk.

14. Reduce 4 yd. 2 ft. 6 in. to the decimal of one mile.

15. A sum of money is divided among A, B, and C.
A gets $\frac{1}{4}$ of it; B $\frac{2}{3}$ of the remainder, and C $\frac{1}{2}$ of what then
remains. There is still left $75, which is equally divided
among the three. Find the total sum received by each.

16. A steam dredge removes 3 cu. ft. of dirt every
minute during a working day of 9 hours. How long a
ditch will this dredge dig in 30 days if the ditch is 12 ft.
wide and $7\frac{1}{2}$ ft. deep?

17. A wall 180 ft. long, 6 ft. high, and 15 in. thick, is
built of concrete made up of 1 part of cement to 8 parts
of gravel. Find the cost of the material if the gravel
costs $1·00 a cubic yard and the cement 50c. a cubic foot.

18. A well-ventilated class-room should supply 2500
cubic feet of pure air for each pupil every hour. How often

in one hour should the air be changed in a class-room 30 ft. long, 24 ft. wide, and $12\frac{1}{2}$ ft. high, if there are 36 pupils in the room?

19. A bushel of wheat takes from the soil $2\cdot4\%$ of its weight in nitrogen, $\cdot9\%$ in phosphorus, and $\cdot6\%$ in potash. A bushel of potatoes takes $\cdot2\%$ of its weight in nitrogen, $\cdot1\%$ in phosphorus, and $\cdot3\%$ in potash. If an acre of ground produces on an average 14 bu. of wheat or 85 bu. of potatoes, find the total weight of plant food removed by each of these crops from 10 acres of ground.

20. If flour is 82% of the wheat used and bread is $133\frac{1}{3}\%$ of the flour used, how many 2-lb. loaves can be made from 75 bu. of wheat?

21. A coal merchant buys coal at $7 a long ton (2240 lb.) and sells it at 8\cdot$50 a short ton (2000 lb.). Find his gain per cent.?

22. On 450 yd. of imported goods invoiced at 2\cdot$40 there was an ad valorem duty of $12\frac{1}{2}\%$ and a specific duty of 5c. a yd. For what amount must the goods be marked to make a profit of $33\frac{1}{3}\%$, after allowing a discount of $16\frac{2}{3}\%$?

23. On a bill of goods amounting to $640, a cash discount of 10% was allowed. The goods were sold at a profit of 25% on their actual cost. Find the gain if 5% of the sales was uncollectible.

24. A city assessed for $13,600,000 builds a bridge costing $300,000. What will a taxpayer who is assessed for $8400 pay toward the bridge?

25. How much money must a man send his agent to buy for him 750 cwt. of sugar at 12c. a pound, to pay the agent's commission of $2\frac{1}{2}\%$, and to pay freight charges at the rate of $10 a ton?

26. An insurance company pays an agent 2% of all premiums collected by him. How much will the company receive from a premium on a house valued at $8000 and its contents valued at $6000 if each is insured for 80% of the value at 90c. on each $100?

27. A man wishes to borrow from his bank $480, and gives a six-month note without interest, which the bank discounts at 8% per annum. What will be the face of the note provided no account is taken of the days of grace?

28. A commission agent sold some mining property on the understanding that he was to receive 5% on the first $25,000 of the sale and 2% on the remaining amount. His total commission was $2750. Find the amount for which the property was sold.

COMPOUND INTEREST

If Mr. Longford borrows from Mr. Sinclair $200 for which he is to pay interest at 5% per annum, he will owe at the end of the year $200 and $10 interest. If, now, he is unable to pay the interest, he is really borrowing $210 from Mr. Sinclair for the second year, the interest on which will be $10·50, and the amount Mr. Longford will owe at the end of two years is $220·50. The difference between this amount and the original principal is called the **compound interest** on that principal for the two years. That is, $20·50 is the compound interest on $200 for 2 years at 5% per annum.

The ordinary interest on $200 for 2 years at 5% is $20, which is called the **simple interest**. In what respect does compound interest differ from simple interest?

The method of finding compound interest is the same as that used for simple interest.

Example: Find the amount of, and the interest on, $1500 for $1\frac{1}{2}$ years at 4% per annum, the interest to be compounded semi-annually.
16

FIRST SOLUTION

$1500 = first principal.

    ·02

    $30 = interest for first ½ year.

    1500

    $1530 = amount after first ½ year.

    ·02

    $30·60 = interest for second ½ year.

    1530·00

    $1560·60 = amount after 1 year.

        ·02

    $31·21 = interest for third ½ year.

    1560·60

    $1591·81 = amount after 1½ years.   Interest = $91·81.

SECOND SOLUTION

Amount after any ½ yr.	= 1·02 × principal at the beginning of that ½ year.
Amount at the end of first ½ yr.	= 1·02 × first principal
	= second principal.
Amount at the end of second ½ yr.	= 1·02 × second principal
	= 1·02 × 1·02 × first principal
	= third principal.
Amount at the end of third ½ yr.	= 1·02 × third principal
	= 1·02 × 1·02 × first principal
	= 1·061208 × $1500
	= $1591·81.
Interest for 1½ yr.	= $1591·81 — $1500
	= $91·81.

Note.—For brevity it is usual to write $1 \cdot 02^2$ instead of $1 \cdot 02 \times 1 \cdot 02$, and $1 \cdot 02^3$ instead of $1 \cdot 02 \times 1 \cdot 02 \times 1 \cdot 02$, and so on. The small figure placed above and to the right of the number indicates the number of times the factor is repeated, thus: $3 \times 3 \times 3 \times 3 \times 3$ is written 3^5.

Since the amount at the $1\frac{1}{2}$ yr. $= 1 \cdot 061208 \times$ principal, what fraction of the principal must the interest be?

What was done in the Second Solution was to find the amount of, or the interest on, $1 for the given time at the given rate, and multiply this by the number of dollars in the principal.

EXERCISE 114

(In these exercises find results to the nearest cent.)

1. Find the amount and the compound interest of $500 for 3 years at 4%. [Interest $= (1 \cdot 04^3 - 1) \times \$500$.]

2. Find the amount and the compound interest of $200 for 2 years at 5%; $3000 for 4 years at 6%; $2000 for 3 years at $5\frac{1}{4}\%$; $4800 for 4 years at $3\frac{1}{2}\%$.

3. Find the amount and the compound interest of $300 for 2 years at 4% per annum compounded half-yearly. (Amount $= 1 \cdot 02^4 \times \$300$.)

4. Find the amount and the compound interest of $800 for $1\frac{1}{2}$ years at $3\frac{1}{2}\%$ per annum compounded half-yearly; $2000 for 1 year at 4% per annum compounded quarterly.

5. Find the amount of $1 in 4 years at 5% compounded yearly. Use the result to find: (a) The amount of $650 for 4 years at 5% compounded yearly; (b) the sum which in 4 years at 5% compounded yearly will amount to $7250.

6. Find the principal which will amount to $3800 in $1\frac{1}{2}$ years at 5% compounded half-yearly; $4750 in 1 year at 4% per annum compounded quarterly.

7. A savings bank which pays 4% per annum compounds its interest half-yearly on June 30th. and December 31st. How much will a man have to his credit on December 31st. who deposits $300 on February 1st. and $900 on August 1st.?

8. Find the difference between the simple interest and the interest compounded half-yearly on $1 for 1½ years at 6%. Use the result to find:

(a) The difference between the simple interest and the interest compounded half-yearly on $900 for 1½ years at 6% per annum.

(b) The sum for which the difference between the simple interest and the interest compounded half-yearly is $28·30 for 1½ years at 6% per annum.

INTEREST TABLES

Banks and other financial institutions calculate interest on their accounts by means of Interest Tables. By way of illustration, the following has been selected from one of these Tables:

TABLE FOR SIMPLE INTEREST

Dollars	67 Days		143 Days		231 Days	
	3½%	4%	5%	5½%	6½%	7%
100	·64·2	·73·4	1·95·9	2·15·4	4·11·4	4·43·0
200	1·28·5	1·46·8	3·91·8	4·31·0	8·22·7	8·86·0
300	1·92·7	2·20·3	5·87·7	6·46·4	12·34·2	13·29·0
400	2·57·0	2·93·7	7·83·6	8·61·9	16·45·5	17·72·1
2100	13·49·2	15·41·9	41·13·7	45·25·1	86·38·8	93·03·3
2200	14·13·4	16·15·3	43·09·6	47·40·6	90·50·1	97·46·3
2300	14·77·7	16·88·8	45·05·5	49·56·0	94·61·5	101·89·3
2400	15·41·9	17·62·2	47·01·4	51·71·5	98·72·9	106·32·3
6500	41·76·1	47·72·6	127·32·9	140·06·1	267·39·1	287·95·9
6600	42·40·3	48·46·0	129·28·8	142·21·6	271·50·4	292·38·9
6700	43·04·5	49·19·5	131·24·7	144·37·2	275·61·8	296·81·9
6800	43·68·8	49·92·9	133·20·5	146·52·6	279·73·1	301·24·9
7300	46·90·0	53·60·0	143·00·0	157·30·0	300·30·0	323·40·0
7400	47·54·2	54·33·4	144·95·9	159·45·5	304·41·4	327·83·0
7500	48·18·5	55·06·8	146·91·8	161·60·9	308·52·7	332·26·0
7600	48·82·7	55·80·3	148·87·7	163·76·4	312·64·2	336·69·0

The Table is self-explanatory. It will be observed that in the numbers ·given in the interest columns there are two decimal points. The part before the first point is to

be read as dollars, the part before the second point as cents, and the last part as a decimal of a cent.

It is clear, too, that when the interest for $7300 is given, the interest for $730, $73, $7·30, 73c. can be found by shifting the decimal point; and that the interest on $7323 can be found by adding the interest on $23 to the interest on $7300.

<div align="center">

EXERCISE 115

</div>

1. Using the above Table, find the interest on $2441·65 for 143 days at $5\frac{1}{2}\%$.

<div align="center">

Interest on $2400	= $51·715
" " 40	= ·8619
" " 1	= ·02154
" " ·65	= ·0140061

" " $2441·65 = $52·61.

</div>

2. What is the interest for 67 days at $3\frac{1}{2}\%$ on $6800? $680? $6·80? $2365? $430?

3. What is the interest for 67 days at 4% on $423? $366? $224·75? $6741·24?

4. What is the interest for 143 days at 5% on $7410? $6524? $2368? $140·20?

5. What is the interest for 143 days at $5\frac{1}{2}\%$ on $7673? $2123? $210? $2·40?

6. What is the interest for 231 days at $6\frac{1}{2}\%$ on 40c.? $6·50? $23·00? $4·75?

7. What is the interest for 231 days at 7% on $24·00? $4000? $68,421? $24,321·40?

<div align="center">

COMPOUND INTEREST TABLES

</div>

The following is an example of a Table used for compound interest. It shows the amount of one dollar at compound interest at the given rates for the specified number of years. The amount of any other number of

dollars will be found by multiplying the amount of one dollar for the given time at the given rate by the number of dollars on which the interest is to be determined.

Yr.	1½%	2%	2½%	3%	3½%
1	1·015000	1·020000	1·025000	1·030000	1·035000
2	1·030225	1·040400	1·050625	1·060900	1·071225
3	1·045678	1·061208	1·076891	1·092727	1·108718
4	1·061364	1·082432	1·103813	1·125509	1·147523
5	1·077284	1·104081	1·131408	1·159274	1·187686
6	1·093443	1·126162	1·159693	1·194052	1·229255
etc.	etc.	etc.	etc.	etc.	etc.

The compound interest on any principal sum for 2 years at 5% per annum payable half-yearly is, of course, the same as the interest on that sum for 4 years at 2½% payable yearly.

<div align="center">EXERCISE 116</div>

1. Find the amount and interest, at 3½% per annum on: $300 for 2 yr.; $400 for 3 yr.; $750 for 5 yr.

2. Find the amount and interest at 2% per annum when the principal and time are $640 for 3 yr.

3. Find the amount and interest at 2½% per annum when the principal and time are: $800 for 3 yr.; $250 for 4 yr.

4. Find the amount and interest at 3% per annum when the principal and time are: $235 for 2 yr.; $480 for 5 yr.

5. Find the principal when the amount, time, and rate are: $1500, 3 yr., 3½%; $2000, 5 yr., 3%.

6. On July 1st, 1920, Mr. A. Brown bought ten D. S. Certificates for which he paid $42·50 each. At the end of three years the Dominion of Canada will redeem these certificates at $50 each. How much more will Mr. Brown then have than he would have had if he had deposited his money in a bank which pays interest half-yearly at the rate of 4% per annum?

STOCKS

It often happens that a business undertaking may be too large for one or two individuals to supply all the money which may be needed, and so a number of persons unite and form what is called a **stock company** or **corporation.** They arrange to raise the money necessary for the business by fixing the **capital** of the company at a settled amount, not of money but of **stock.** This stock is then divided up into an equal number of shares, and each share is valued at a fixed amount, usually $100, which is called the **par value** of a share of stock. These shares are then sold.

A person becomes a member of the company by buying one or more of the shares of its stock. He is then a **stockholder,** or **shareholder,** and is given a stock certificate.

The **profits** of the company are called its **dividend** and are divided at regular periods among the shareholders according to the number of shares each possesses.

Stock is not money, but it can be bought and sold for money, and a shareholder can get money for his stock only by selling it to some person who is willing to buy. The price paid for it will depend largely upon the success of the company and upon the amount of the dividend.

Although, as stated above, stock is usually divided into $100 shares, there are stocks with shares of different amounts, such as $1, $5, $10, $50, $200.

With the exception perhaps of some mining stocks, the market value of the shares is quoted in stock-lists, newspapers, and elsewhere, as *a percentage of the par value*. Thus, Bank of Toronto stock listed at 193 means that a $100 share sells for $193.

Stock is at par, at a discount, or at a premium, according as its shares sell for, below, or above their par values. The market value of stock must be clearly distinguished from its par value.

Stock is generally bought and sold through a **stock-broker.** He charges for his services a percentage of the par value of the stock. This charge is called **brokerage.**

EXERCISE 117

1. What is the par value and the market value of 25 shares of Dominion Bank stock at 241? of 60 shares of Dominion Steel stock at $74\frac{1}{4}$? of 57 shares of Toronto Street Railway at $63\frac{1}{4}$? of 23 shares of Lake Superior at 27?

2. How many shares can be bought of Twin City Railway stock at $56\frac{1}{2}$ for $3,390? of Imperial Bank stock at 230 for $9,430? of Mexican Light and Power stock at $66\frac{1}{2}$ for $3,657·50?

3. What is the annual dividend obtained from 36 shares of stock which pays 6%? from 92 shares of stock which pays $12\frac{1}{2}$%? from 44 shares of stock which pays $3\frac{1}{2}$% half-yearly?

4. How many shares of the par value of $100 will there be in a total capital of $50,000 stock? of $362,500 stock?

5. What will be the dividend on each share of stock when the capital and total profits are, respectively, $250,000 and $11,250? $1,500,000 and $123,750?

6. What is the rate of the dividend when:

(a) $11,000 stock gives an annual income of $495?

(b) $9000 stock gives an annual income of $382·50?

(c) $7500 stock gives an annual income of $400?

7. The holder of 58 shares of railway stock sells at 65 and invests the proceeds in a bank stock at 145. How many shares of bank stock does he buy?

8. A man invests $6150 in a bank stock at 123. If the stock pays a dividend of 7% per annum, what is the man's income from his stock?

9. Mr. Thornton instructs a broker to buy for him 25 shares of Lake of the Woods stock at 135¾. If the broker charges ¼% commission, what does the stock cost Mr. Thornton? How much does the broker receive?

10. I instruct my broker to sell for me 18 shares of Nova Scotia Coal stock at 69. If he charges ½% brokerage, what do I receive for the stock?

11. A man invests $3,780 in stock at 94¼ and sells when the stock rises to 110¾. How much does he gain by the transaction if he has to pay ¼% brokerage for buying and the same for selling?

12. What annual income will be obtained from investing $10,010 in a 6% stock at 125, brokerage being ⅛%?

13. The holder of $7,200 stock which pays an annual dividend of 5% sells at 96½ and invests the proceeds in an 8% stock at 119½. What will be the difference in his annual income, brokerage for buying and selling being ½% in each case?

14. A man invests $3,820 in a 3% stock at 95½, $6,000 in a 4% stock at par, and $8,660 in a 5% stock at 108¼. How many shares of stock has he? What is their total par value? What is his total income from his stocks?

15. Mr. Riddell invests $4,800 in a 6% stock at 120. How many shares of stock does he buy? What will be his annual income? What fraction is this income of the money invested? What per cent., therefore, is the rate of interest on the money invested?

16. If I invest $5000 in an 8% stock at 125, what rate of interest do I receive on the money invested?

17. If I pay $75 for a share of 6% stock, what rate of interest shall I get on the money invested?

18. A man has $4200 which he can loan at 5½% per annum or invest in a 6% stock at 105. How much larger will his income be by taking the better investment?

19. How much money must be invested in a $7\frac{1}{2}\%$ stock at 115 to give an annual income of $300?

20. How much 4% stock must I have to give me an income of $360? What would it cost at 80?

21. A man who invests in a 10% stock finds that he is getting $6\frac{1}{4}\%$ interest on the money invested. What is the price of the stock?

EXCHANGE

The buying and selling of goods is constantly going on between all parts of the world, and, as a result, it is necessary to have some means by which the payment of money can be made between places distant from each other.

The settling of accounts between persons residing at a distance from each other by means of written orders is called **exchange.** These written orders, such as cheques, drafts, express money-orders, etc., are called **bills of exchange.**

If Mr. James Kay, living in Peterborough, owes $50 to Mr. William Wye, living in Ottawa, in what different ways may Mr. Kay pay his debt?

Paying by cheque: In the example above, if Mr. Kay has money to his credit in any bank, he may make out a cheque for $50. Write the cheque.

This cheque he will send to Mr. Wye, who can get it cashed at some bank in Ottawa. The bank will probably deduct a small sum from the face of the cheque.

Paying by bank draft: Instead of sending the above cheque, Mr. Kay might go to a bank and buy a **bank draft.**

This draft is sent by Mr. Kay to Mr. Wye, who gets it cashed for its full value in Ottawa. Mr. Kay would probably have to pay for the draft a little more than $50.

Commercial draft: The account between Mr. Kay and Mr. Wye may also be settled by Mr. Wye making a draft on Mr. Kay. Write the draft.

Some bank in Ottawa pays Mr. Wye on this draft $50 less the cost of exchange, and sends the draft to a bank in Peterborough, which collects the money from Mr. Kay.

The sum to be paid for drafts or bills of exchange is usually a certain per cent. of their face value, and the difference in percentage between the face value and the amount paid is called the **rate of exchange.**

Post-office, express, and **bank money-orders** are in form much like bank drafts. They are bought and cashed at post-offices, Express Companies' offices, and at banks, respectively. They are sold for a slight advance over their face values.

In the case of Post-office Orders issued in Canada for payment in Canada and United States, the advance above the face value is as follows:

For $5 and under, 3c. Over $30 and up to $50, 15c.
Over $5 and up to $10, 6c. Over $50 and up to $75, 25c.
Over $10 and up to $30, 10c. Over $75 and up to $100, 30c.

For small sums the post-office also sells **postal notes** of the following denominations:

1. 20c., 25c., 30c., 40c., each of which costs 1 cent extra.

2. 50c., 60c., 70c., 80c., 90c., $1, $1·50, $2, and $2·50, each of which costs 2 cents extra.

3. $3, $4, $5, and $10, each of which costs 3 cents extra.

The odd cents are made up by attaching postage-stamps to the note; for instance, 38 cents would be made up by a 30c. postal note and 8c. in postage-stamps.

The extra charges for express and bank money-orders up to $50 are the same as those for Post-office Orders.

In case of Foreign Exchange, that is, exchange between two places which are in different countries, we shall have to consider, not only the rates of exchange, but also the value of the coins used in the two places.

The normal values of the coins of a few countries are here given:

Country	Coin	Canadian Value
Great Britain........	1 pound..............	$4·86¾
France............ Belgium............ Switzerland........	1 franc (100 centimes)	19·3 cents
Germany...........	1 mark (100 pfennige)	23·85 cents

EXERCISE 118

1. At the rates quoted above, find the cost of post-office money-orders for: $4·10, $14, $26·50, $30·08, $47·75, $74·99, $93·30.

2. For each of the following sums what postal notes would you buy, and what would be the total cost in each case: 45c., 95c., $1·65, $2·93, $3·79, $4·85, $5·92, $26?

3. What is the difference in cost between money-orders and postal notes for: 85c., $3·30, $6·40, $18, $30, $40·15?

4. The rate of exchange being ⅛%, find the cost of a bank draft for $360, for $2000, for $236·50.

5. Mr. James Wilkinson, St. Thomas, buys $1200 worth of goods from Brown Bros., Toronto, and sends them a draft, issued by the Merchants Bank, for the

amount. What will this draft cost Mr. Wilkinson, exchange being at $\frac{1}{8}\%$?

6. The Atlantic Lumber Co., Halifax, sold $2400 worth of lumber to R. Higgins & Co., Montreal, and drew on them for the amount through the Bank of Nova Scotia. How much would the Lumber Company get from the bank, exchange being at $\frac{1}{4}\%$?

7. F. Davis of Simcoe sends Mr. R. Carter, Belleville, a cheque on the Bank of Hamilton in payment of an account of $160. Write the cheque. What will Mr. Carter get for it, exchange being at $\frac{1}{8}\%$?

8. What will a Toronto merchant pay for a draft on Chicago for $3200, exchange being at $\frac{1}{4}\%$?

9. What will be the face of a draft for which $720·90 is paid, when exchange is at $\frac{1}{8}\%$?

10. What must be the face of a cheque, so that when it is cashed at a bank the proceeds will be sufficient to pay a bill for $319·20, the bank's charge for exchange being $\frac{1}{4}\%$?

11. What must be the face of a cheque, so that when the bank's charges of $\frac{1}{2}\%$ for exchange are deducted, the proceeds will be sufficient to pay a bill for $139·30?

12. When $1565·20 is paid for a $1560 draft, what is the rate of exchange?

13. What is the value in Canadian money of: £1240, 3500 francs, 4800 marks?

14. $6400 is equal to how many pounds? $4530 to how many francs? $8000 to how many marks?

SQUARE ROOT

When a number is multiplied by itself, the product is called the **square** of the number, and the number is called the **square root** of the product. For example, 16 is the square of 4, and 4 is the square root of 16.

It is evident that, when a number can be factored and its factors arranged in two groups each containing the same factors, the product of the factors in one of these groups is the square root of the number.

Example: $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= (2 \times 3 \times 3) \times (2 \times 3 \times 3).$

The square root of 324 is $2 \times 3 \times 3$, or 18.

EXERCISE 119

1. What are the squares of: 3, 4, 8, 9, 10, 7, and 6?

2. What are the squares of: 12, 15, 18, 45, and 90?

3. What are the square roots of: 9, 16, 49, 121, 1600, and 8100?

4. What are the square roots of: $\cdot 25$, $\frac{1}{36}$, $\cdot 49$, $1 \cdot 44$, and $\frac{64}{169}$?

5. What are the lengths of the sides of squares whose areas are: 64 sq. in., $\cdot 09$ sq. yd., 3600 sq. rd., $\frac{1}{4}$ sq. mi.?

6. What is the perimeter of each of the squares whose area is given in Question (5)?

7. Find, by factoring, the square root of: 324, 1089, 7569, 9801, 15625, 65536, $\cdot 2601$, $125 \cdot 44$, 527076.

8. A lot is 81 ft. wide and 169 ft. long. What is the length of the side of a square that has the same area?

9. The width of a lot is $\frac{3}{4}$ of its length. If the lot contains 30 acres, find its perimeter.
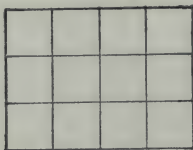
The numbers 9, 64, 169, and 625, are perfect squares, but the numbers 10, 27, 85, and 113 are not. Why?

Square the numbers 1 and 9, 10 and 99, 100 and 999, 1000 and 9999, and the decimals ·1 and ·9, ·01 and ·99, ·001 and ·999. From the results it will be seen that there are twice as many, or 1 less than twice as many digits in the square as in the number itself. Hence, if the digits of a number be separated into periods of two digits each, beginning at the decimal point, the number of periods will be the same as the number of digits in the square root of the number. Thus, the square root of:

5/59/32/25 will have 4 integral places, and the square root of: 41/43/·49/69 will have 2 integral and 2 decimal places.

If this is known, the value of the highest digit in the square root of any number can be found by inspection.

Example: What is the highest digit in the square root of 4761.

Separating the digits into periods, we get 47/61, which shows that there will be 2 places in the square root.

Therefore, the highest digit in the square root is a tens' digit. Now 6 tens, or 60, is the greatest ten whose square is contained in 47 hundred. Hence, the ten's digit in the square root is 6.

ORAL EXERCISE

1. How many digits will there be in the squares of integral numbers containing 3, 4, 5, 6, 7 digits, respectively?

2. How many digits will there be in the squares of numbers containing 1 integral and 1 decimal place? 2 integral and 1 decimal place? 1 integral and 2 decimal places? 3 integral and 4 decimal places?

3. How many digits will there be in the square roots of numbers containing 3, 4, 5, 7, or 10 integral places?

4. How many digits will there be in the square roots of numbers containing 3, 4, 5, 7, or 8 decimal places?

5. How many digits will there be in the square roots of numbers containing 3 integral and 1 decimal place,

4 integral and 3 decimal places, 5 integral and 5 decimal places?

6. What will be the value and the place of the highest digit in the square roots of: 265, 3708, 69214, ·2618, 432·15?

When a number is not easily factored, or when the factors cannot be arranged into two equal groups, the square root is found as follows:

Example: Find to one decimal place the square root of 3246·5.

$$
\begin{array}{r}
3246 \cdot 50(56 \cdot 9 \\
25 \\
\hline
106)746 \\
636 \\
\hline
112 \cdot 9)110 \cdot 50 \\
101 \cdot 61 \\
\hline
8 \cdot 89
\end{array}
$$

Explanation:

The steps in the method are as follows:

1. Begin at the decimal point and separate the digits, left and right, into periods of two figures each.

2. Find the greatest number whose square is contained in the left-hand period, place it at the right as if it were the first digit of a quotient, subtract its square from the left-hand period, and to the right of the remainder add the next period for a dividend.

3. Double the part of the square root already found, and place the product at the left for a trial divisor; divide the dividend, leaving out the right-hand digit, by this trial divisor, and the quotient will be the second digit in the square root.

4. Place the second digit to the right of the trial divisor, and multiply the whole divisor thus formed by this second digit; subtract the product from the divi-

dend; and to the right of the remainder place the next period to get the next dividend.

5. Again double the part of the square root now found; use the product for a trial divisor, and find the third digit of the square root as before. So proceed until all the periods are taken down.

Find the square root of each of the following numbers:

1. 3844, 4096, 5329, 8464.

2. 19881, 24336, 37249, 65536, 97969.

3. 173056, 277729, 356409, 654481, 772641.

4. 2819041, 7387524, 38576521.

5. ·1296, 3249, ·6241, ·7744, ·0289, ·0729.

6. 7·29, 11·56, 146·41, 49·1401, 341·1409.

7. ·150544, ·327184, 1218·7081, 5·774409.

8. Find the square roots of the following, correct to three places of decimals:

$$2, 3, 5, 10, 17, 123, ·9, ·51, 6·2, 4·25.$$

9. Find the square roots of the following fractions:

$$\tfrac{881}{576}, \tfrac{784}{841}, \tfrac{1444}{2401}, 3\tfrac{1}{16}, 25\tfrac{121}{144}, 32\tfrac{164}{625}.$$

10. Find to three places of decimals the square root of each of the following:

$$\tfrac{3}{4}, \tfrac{5}{7}, \tfrac{3}{11}, \tfrac{4}{27}, 4\tfrac{1}{2}, 3\tfrac{2}{3}.$$

11. At what rate per cent. per annum will a sum of money amount to 1·1025 of itself in two years if interest is compounded annually?

12. A rectangular field is 45 rods long and 25 rods wide. Find the length of the side of a square field whose area is the same as that of a rectangular one.
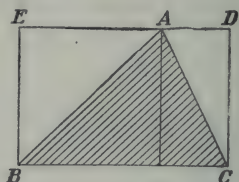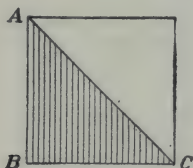
SECTION VII

MEASUREMENTS

1. PERIMETERS AND AREAS

(1) *The Rectangle*

For measurements of **rectangles** and **squares** see pages 122-124.

(2) *The Triangle*

The area of a triangle = ½ *the area of a rectangle which has the same altitude and length of base as the triangle.*

EXERCISE 121

1. Find the area of a rectangle whose base is 15 ft. and whose altitude is 8 ft. What will be the area of a triangle having the same base and altitude as the rectangle?

2. Find the area of a triangle whose base is 22 ft. and whose altitude is 11 ft. What will be the area of a rectangle having the same base and altitude as the triangle?

3. Find the area of a triangle whose base is 13 ft. and whose altitude is 7 ft. 4 in.

4. Find the number of feet of inch lumber which will be required to make a triangular floor having a base 18 ft. long and an altitude of 14 ft. 2 in.

5. Look at the walls of your house or barn. What parts of them are triangles? Get the measurements of these triangles. Find how many feet of inch lumber it

will require to cover the gable end of the barn; also how much it will cost to paint it at 15c. a sq. yd. How much inch siding will it require for the four walls of your house or barn? How much will it cost to paint the barn?

6. The area of a triangle is 12 sq. yd.; its base is 9 ft. Find its altitude.

7. The area of a triangle is 135 sq. in.; its altitude is 1 ft. 6 in. Find the length of its base.

(3) The Right-angled Triangle

In the **right-angled triangle** the side opposite the right angle is called the **hypotenuse.** What names have been given to the other two sides?
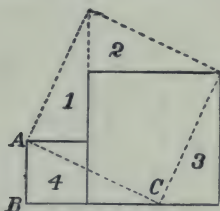
On each of the sides of a right-angled triangle, ABC, construct a square. Place the two small squares as shown by the heavy lines and the large square as shown by the dotted lines in the figure. You will see by placing the triangle (1) on the triangle (4) and the triangle (2) on the triangle (3) that:

The area of the square on the hypotenuse = the sum of the areas of the squares on the other two sides.

EXERCISE 122

1. The base of a right-angled triangle is 4 ft., the altitude is 3 ft. What is the area of the square on the hypotenuse? What is the length of the hypotenuse? Construct the triangle and measure the sides.

2. Find the length of the hypotenuse of a right-angled triangle whose base is 12 ft. and whose altitude is 9 ft.

3. A wall is 15 ft. high and 20 ft. long. Find the distance from the upper right-hand corner to the lower left-hand corner.

4. Find the length of wire which will reach from the top of a telegraph pole 20 ft. high to a point on the ground 24 ft. from the foot of the pole.

5. The hypotenuse of a right-angled triangle is 16 ft. and the base is 6 ft. Find the altitude, correct to 2 places of decimals.

6. Mention lines in your school-room which will be the sides of right-angled triangles. Measure two of the lines for any triangle and then calculate the length of the third. Check your answers by measuring the lines.

(4) *The Circle*

(a) **Circumference.**—What is meant by the diameter of a circle? By the radius? By the circumference? Mention some objects about you whose surfaces are circles. Measure their diameters and their circumferences. By division find, to two decimal places, the number of times the circumference contains the diameter.
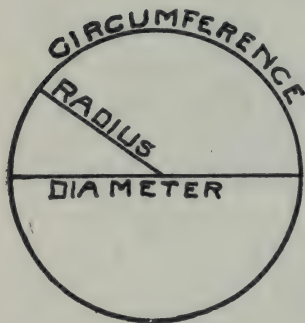
It will be found that approximately:

The circumference of a circle = 3⅐ times its diameter.

EXERCISE 123

1. What is the circumference of a circle whose diameter is 14 ft? 8 ft.? 6 ft.? 20 in.?

2. What is the circumference of a circle whose radius is 6½ ft.? 9 ft.? 12 ft.?

3. What is the diameter of a circle whose circumference is 33 ft.? 18 ft.? 75 ft.?

4. A wheel 4 ft. in diameter will make how many turns in going 2 miles?

5. The diameter of a circular garden is 21 yd. How much farther will a man have to walk to reach the opposite side, if instead of crossing the garden he goes around it?

6. A locomotive runs at the rate of 42 miles an hour. The diameter of the driving-wheel is 6 ft. How many turns does the wheel make in a minute? in a second?

(b) **Area.**—Cut out a circle of leather or stiff paper.

Divide it into halves and place one half on top of the other. Now, by cutting from the centre of the circle to, *but not through*, the circumference, divide the two halves into the same number of triangular shapes as small as possible. Then open out each half and insert the parts of one half between those of the other, as shown in the figure below.
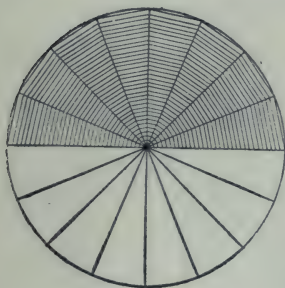

It will be seen that, if there are parts enough, the figure will become a rectangle.

The area of a circle is equal to ½ the area of a rectangle of which the base is equal to the circumference and the altitude equal to the radius of the circle.

EXERCISE 124

1. Find the area of a circle whose radius is 7 ft., 5 ft. 3 in., 19 yd., 24 ft. 6 in.

2. Find the area of a circle whose diameter is 8 ft., 15 ft., 10 ft. 2 in., 12 yd.

3. Find the area of a circle whose circumference is 22 ft., 33 yd., 40 rd.

4. How many acres of land will be inclosed by a circular race-track 1 mile long?

5. Find in yards, correct to two decimal places, the circumference of a circular garden containing $\frac{1}{4}$ of an acre.

6. Find the area of a circle whose diameter is 14 feet. Give dimensions for rectangles having the same area as the circle.

7. Measure the diameter of a 25-cent piece, of a 50-cent piece. Find the area of each.

(5) *The Cylinder*
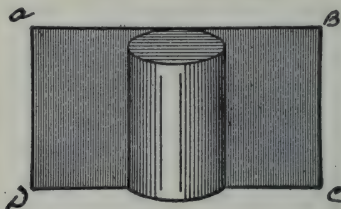
How many faces has a cylinder? Name objects which are cylindrical in shape. Is your pencil a cylinder? Is a 5-cent piece a cylinder?

How is the area of the two ends of a cylinder found? If a piece of paper be fitted about the curved face of a cylinder and then unrolled, as shown in the foregoing Figure, what would be the shape of the paper? How is its area found?

The area of the curved face of a cylinder equals that of a rectangle of which the base is equal to the circumference and the altitude equal to the height of the cylinder.

1. Find the area of the curved surface of a cylinder whose circumference is 10 ft. and altitude is 8 ft.; circumference $15\frac{1}{2}$ ft. and altitude $5\frac{1}{2}$ ft.; circumference 3 ft. and altitude 18 in.

2. Find the area of the curved surface of a cylinder, of which the altitude is 20 ft. and the diameter is 4 ft. 8 in.

3. What will it cost at $1\frac{1}{2}$ c. a sq. ft. to paint the outside of a cylindrical silo the radius of which is $6\frac{2}{3}$ ft. and whose altitude is 21 ft.?

4. A lawn roller is 3 ft. 6 in. long and 21 in. in diameter. How many square yards of lawn will it cover in making 225 turns?

5. Find the difference, in square inches, between the total area of the surface of a cube 7 in. long and that of a cylinder 7 in. in diameter and 7 in. in height.

6. The area of the curved face of a cylinder is 396 sq. ft. The altitude is 12 ft. Find the diameter.

7. Find the area of the curved surface of a cylinder the base of which contains $9\frac{5}{8}$ sq. in. and the height of which is 16 in.

2. VOLUMES

(1) *Prisms and Cylinders*
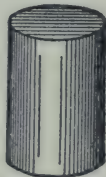
We have here the figures of three solids. The first is known as a **rectangular** (in this case **square**) **prism;** the second is a **triangular prism;** and the third is a **cylinder.**

How is the volume of the rectangular prism found? How can the volume of the triangular prism be found? In both cases, *the number of cubic units in the volume equals the number of square units in the area of the base multiplied by the number of linear units in the height.* The same is true of the cylinder.

EXERCISE 126

1. What is the volume of a square prism whose altitude is 15 ft. 6 in. and the side of whose base is 4 ft.? Find also the area of its lateral surface.

2. What is the volume of a triangular prism 30 ft. high, one side of whose base is 8 ft., and the length of the perpendicular on it from the opposite angle is 5 ft.?

3. What is the volume of a cylinder whose diameter is 12 ft. and whose height is 32 ft.?

4. How many cubic feet of water will a circular cistern hold if the diameter is 6 ft. and the depth $10\frac{1}{2}$ ft.? How many gallons will it hold, there being $6\frac{1}{4}$ gal. in 1 cu. ft.?

5. The water from a flat roof is carried into a rectangular cistern 6 ft. square. If the whole roof is 36 ft. long and 28 ft. wide, how high will a rainfall of $\frac{1}{4}$ in. raise the water in the cistern?

EXERCISE 127 (REVIEW)

1. A flagpole 48 ft. high casts a shadow 64 ft. long. How far is it, in a straight line, from the top of the pole to the end of the shadow?

2. If there are $1\frac{1}{4}$ bu. in a cubic foot, how many bushels of grain can be stored in a circular tank whose diameter is 30 ft. and whose height is 63 ft.?

3. If a ton of coal measures 40 cu. ft., how many tons can be stored in a rectangular coal shed 60 ft. 4 in. long, 24 ft. wide, and 10 ft. high?

4. There are $6\frac{1}{4}$ gal. in a cu. ft. In 20 min. how many gallons of water will flow through a $3\frac{1}{2}$-in. pipe if the water is running at the rate of 3 mi. an hour?

5. Find the difference in rods, correct to two decimal places, between the perimeter of a square field containing 10 acres and the circumference of a circular field of the same area.

6. How many square inches of cardboard will it take to make a triangular prism 18 in. long, the end of which is a right-angled triangle whose base is 8 in. and whose altitude is 6 in.? What will be the volume of the prism?

7. How many cubic inches of lead will it take to make 3 ft. of $2\frac{1}{2}$-in. pipe, the lead being $\frac{1}{4}$ in. in thickness?

8. Find the total surface area and the volume of a cylinder whose diameter is 14 in. and whose altitude is 30 in.

9. A church steeple 72 ft. 6 in. high casts a shadow 58 ft. long. What will be the height of a flagpole which at the same time casts a shadow 64 ft. 8 in. long?

10. Using a scale $\frac{1}{8}$ of an inch to a foot, draw on a sheet of paper a plan of the whole of the ground floor of your house. At present prices, find the cost of 1-in. lumber which would be required to make the flooring.

11. A silo should provide a base area of 5 sq. ft. for each cow to be fed therefrom. If a ton of ensilage occupies 50 cu. ft., find the diameter and the height of a round silo for 30 cows, each cow to get 40 lb. daily for 180 days.

SECTION VIII

TABLES OF WEIGHTS AND MEASURES

VALUE

1. **Canadian Money:**

>   The standard unit of value is the dollar.

> | 10 mills | = 1 cent (c.) |
> |---|---|
> | 100 cents | = 1 dollar ($) |

2. **United States Money:**

> | 10 mills | = 1 cent |
> |---|---|
> | 10 cents | = 1 dime |
> | 10 dimes | = 1 dollar |
> | 10 dollars | = 1 eagle. |

3. **British Money:**

> | 4 farthings | = 1 penny (d.) |
> |---|---|
> | 12 pence | = 1 shilling (s.) |
> | 20 shillings | = 1 pound (£) |
> | | or 1 sovereign |
> | 5 shillings | = 1 crown |
> | 21 shillings | = 1 guinea. |

£1 = $4·86⅔.

LENGTH

>   The standard unit of length is the yard.

> | 12 inches (in.) | = 1 foot (ft.) |
> |---|---|
> | 3 feet | = 1 yard (yd.) |
> | 5½ yards | = 1 rod (rd.) |
> | 320 rods | = 1 mile (mi.) |

>   1 mi. = 320 rd. = 1760 yd. = 5280 ft.
>   6 ft. = 1 fathom; 4 in. = 1 hand
>   1 mi. = 8 furlongs; 1 knot = 1·15 mi. (nearly)
>   1 chain = 100 links = 22 yd.

AREA OF SURFACE

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (ac.)
640 acres	= 1 square mile (sq. mi.)

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10 square chains	= 1 acre
1 square mile	= 1 section
36 square miles	= 1 township
100 square feet	= 1 square of roofing, flooring, etc.

} In Western Canada

VOLUME

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord (cd.)

CAPACITY

The standard unit of capacity is the *gallon*, which is equal to the space occupied by ten pounds of distilled water at a temperature of 62° F., and which contains 277·274 cubic inches.

1. Dry Measure:

2 pints (pt.)	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
2 gallons	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)
8 bushels	= 1 quarter (British market)
1 bushel	= 2218·192 cu. in.

2. Liquid Measure:

4 gills	= 1 pint
2 pints	= 1 quart
4 quarts	= 1 gallon
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
63 gallons	= 1 hogshead (Hgd.)

1 cu. ft. of water contains almost 6¼ gal. and weighs nearly 62½ pounds.

3. Apothecaries' Fluid Measure:

Used in filling medical prescriptions

60 minims (m.)	= 1 fluid drachm ($f\mathfrak{Z}$)
8 fluid drachms	= 1 fluid ounce ($f\mathfrak{Z}$)
18 fluid ounces	= 1 pint (O)
8 pints	= 1 gallon.

WEIGHT

The standard unit of weight is the pound avoirdupois.

1. Avoirdupois Weight:

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.) or 1 cental
2000 pounds	= 1 ton (T.)

---

196 pounds	= 1 barrel of flour
200 pounds	= 1 barrel of beef or pork
280 pounds	= 1 barrel of salt
14 pounds	= 1 stone
2240 pounds	= 1 long ton, used sometimes to measure coal, iron, etc.

2. Troy Weight:

24 grains (gr.)	= 1 pennyweight (dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

---

1 pound Troy = 5760 grains
1 pound Avoirdupois = 7000 grains.

The unit used for weighing diamonds, gems, etc., is the *carat*, which is about 3·2 grains Troy.

The carat is also used to express the fineness of gold. For instance, 18-carat gold is $\frac{18}{24}$ pure gold, and 10-carat gold is $\frac{10}{24}$ pure gold.

3. Apothecaries' Weight:

    Used in filling medical prescriptions

    20 grains   = 1 scruple (sc. or ℈)
     3 scruples = 1 drachm (dr. or ℨ)
     8 drachms = 1 ounce (oz. or ℥)
    12 ounces = 1 pound (lb.)

MISCELLANEOUS WEIGHTS

Grains, Vegetables, Etc.

60 lb. = 1 bu. of wheat, beans, clover-seed, peas, potatoes
56 lb. = 1 bu. of Indian corn, rye, flax-seed
50 lb. = 1 bu. of onions, beets, carrots, turnips
48 lb. = 1 bu. of barley, buckwheat, timothy-seed
45 lb. = 1 bu. parsnips
44 lb. = 1 bu. of hemp seed
40 lb. = 1 bu. of castor beans
36 lb. = 1 bu. of malt
34 lb. = 1 bu. of oats
14 lb. = 1 bu. of blue grass seed.

TIME

The standard unit of time is the *Mean Solar Day.*

    60 seconds (sec.) = 1 minute (min.)
    60 minutes   = 1 hour (hr.)
    24 hours   = 1 day (dy.)
     7 days   = 1 week (wk.)
    12 months   = 1 year (yr.)
    365 days   = 1 common year
    366 days   = 1 leap year
    10 years   = 1 decade
    100 years   = 1 century.

The true solar year is 365 dy., 5 hr., 48 min., 46 sec.

Every year the number of which is exactly divisible by 4 is a leap year, except those years the numbers for which end in two ciphers. These are leap years only when the numbers are exactly divisible by 400.

ANGLES AND ARCS

Used for angles, direction, latitude, longitude, etc.

60 seconds ('')	= 1 minute (')
60 minutes (')	= 1 degree (°)
360 degrees	= 1 circle or circumference
90 degrees	= 1 quadrant or right angle.

MISCELLANEOUS

Counting

12 articles	= 1 dozen (doz.)
12 dozen	= 1 gross (gro.)
12 gross	= 1 great gross
20 things	= 1 score.

---

24 sheets of paper	= 1 quire
20 quires	= 1 ream.

Paper is frequently sold by the pound or in pads or parcels of 100, 500, or 1000 sheets.

FOREIGN COINS

Normal Values

Coin	Country	Value
Rouble	Russia	$0·515
Franc	France	0·193
Lira	Italy	0·193
Mark	Germany	0·238
Yen	Japan	0·498
Peseta	Spain	0·193
Peso	Chile	0·365
Crown	Sweden	0·268

THE METRIC SYSTEM

The following relations selected from the Metric Tables of weights and measures will be found useful:

1. *Length.* The unit of length is the **metre,** which is equal to 39·37 inches.

$$
\begin{aligned}
10 \text{ millimetres (mm.)} &= 1 \text{ centimetre} \\
10 \text{ centimetres (cm.)} &= 1 \text{ decimetre} \\
10 \text{ decimetres (dm.)} &= 1 \text{ metre} \\
1000 \text{ metres (m.)} &= 1 \text{ Kilometre (Km.)}
\end{aligned}
$$

Approximately, 8 Kilometres = 5 miles.

2. *Volume.*

$$
\begin{aligned}
1000 \text{ cubic millimetres} &= 1 \text{ cubic centimetre} \\
1000 \text{ cubic centimetres} &= 1 \text{ cubic decimetre} \\
1000 \text{ cubic decimetres} &= 1 \text{ cubic metre.}
\end{aligned}
$$

Approximately, 1 cubic centimetre = ·06103 cubic inches.

3. *Capacity.* The unit of capacity is the **litre.** It is equal to a cubic decimetre, and is nearly equal to $1\frac{3}{4}$ pints.

4. *Weight.* The unit of weight is the **gramme.** The gramme is the weight of a cubic centimetre of pure water at its greatest density.

$$
\begin{aligned}
1000 \text{ grammes} &= 1 \text{ kilogramme} \\
1000 \text{ kilogrammes} &= 1 \text{ metric ton.}
\end{aligned}
$$

A kilogramme is about $2\frac{1}{5}$ lb., and a metric ton about 2204·6 lb.

SECTION IX

MISCELLANEOUS PROBLEMS

EXERCISE 128

1. Fill in the following statement of seven weeks' cash receipts, and prove the correctness of your work by adding horizontally and vertically:

Week	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total
1st....	$65·91	$88·74	$41·82	$33·60	$44·18	$29·41	
2nd....	54·36	70·29	36·42	84·72	63·18	98·72	
3rd....	81·27	29·82	26·71	21·90	29·16	23·18	
4th....	54·77	86·41	21·86	90·82	80·27	36·87	
5th....	81·29	43·81	87·90	21·44	24·33	20·09	
6th....	29·04	63·83	27·60	86·92	36·81	74·71	
7th....	40·01	28·42	72·89	27·34	27·42	64·13	
Total							

2. A grocer has a tub of butter on which he would gain $2·24 if he sold it at 64c. a pound, but he would lose $1·68 if he sold it at 57c. a pound. Find the weight of the butter and the cost of one pound.

3. (a) Find the value of: $(36 \times 217 \times 528 \times 75) \div (84 \times 396 \times 155 \times 27)$.

(b) Divide 4682973 by 378, using the factors 6, 7, and 9, and show how to get the complete remainder.

4. A man owing $9260 is able to pay only 48c. on the dollar. What is his property worth?

5. Pure milk is worth 7c. a quart. If 2 quarts of water be mixed with every 3 gallons of milk, what will a quart of the mixture be worth?

6. How many bushels of barley worth 62c. a bushel must a dealer mix with 135 bushels worth 53c. a bushel, in order to make a mixture worth 59c. a bushel?

7. Find the cost of the poles and wire required for a telephone line 9 miles long, measured from the first

to the last pole, if the poles are placed 44 yd. apart and cost $1·10 each and if there are two wires each weighing 180 lb. to the mile and costing $4\frac{1}{2}$c. a pound.

8. Mrs. Libby has a flock of 60 hens, each of which lays on an average 100 eggs a year. All the eggs but 25 doz. are sold at 55c. a dozen. From the 25 doz. are hatched 270 chickens which are sold at 20c. each. Find the net profits for the year if it costs 20c. a month to feed each hen.

9. The product of four consecutive numbers is 143640. Find the numbers.

10. Find the G. C. M. of: 3127, 3551, and 3975.

11. Find the L. C. M. of: 1185, 1580, and 1975.

12. Find the length of the longest unit that will exactly divide 88 yd. 2 ft. 5 in.; and 119 yd. 2 ft. 1 in.

13. A boy on being asked what $\frac{12}{13}$ of a certain fraction was, made the mistake of dividing the fraction by $\frac{12}{13}$, and so got an answer which exceeded the correct answer by $\frac{25}{162}$. What was the correct answer?

14. Find the value of:

(a) $(4\frac{5}{8} + 1\frac{11}{45} + 5\frac{1}{13}) - (2\frac{1}{3} \div \frac{2}{7}) \times 1\frac{2}{7} + 1\frac{3}{8}$.

(b) $1\frac{1}{7} - \frac{3}{4}$ of $(\frac{1}{15} + \frac{1}{5}) + \frac{3}{104} \div \dfrac{2\frac{3}{16}}{3 - \frac{4}{5}}$.

(c) $(7\frac{1}{8} + 5\frac{6}{7} - 6\frac{1}{4}) \div (\frac{1}{7}$ of $8\frac{2}{3})$ of $(7\frac{1}{5} \div 8\frac{2}{8})$.

(d) $(2\frac{2}{9} + 3\frac{11}{12} + 4\frac{3}{8} + 2\frac{5}{9}) \div (3\frac{7}{18} - 2\frac{5}{18})$.

15. If a boy buys peaches at the rate of 5 for 2c. and sells them at the rate of 4 for 3c., how many must he buy and sell to make a profit of $4.20?

16. A merchant bought 3 boxes of soap, each containing 112 lb. He kept the soap all summer, and it lost $\frac{1}{4}$ of its weight. He then sold it at 13c. a lb. Find his gain if the soap cost him 25c. for 3 lb.

17. A man spent $\frac{1}{3}$ of his money, then $\frac{1}{7}$ more than $\frac{5}{8}$ of the remainder. He then found out that he had still $16 left. How much money had he at first?

18. Find the cost of making a road 4 mi. 120 rd. 11 yd. long at $760 a mile.

19. A Canadian dealer imported 210 yd. of tweed at 6s. 3d. a yd. Find the cost in Canadian money if £1 = $4·86⅔.

20. For every cent which A gets, B gets 2½c.; and for every dollar B gets, C gets 25c. If among them they get $132·00, what is each man's share?

21. A farmer sells a merchant 6 loads of potatoes, weighing with the wagon 3567 lb., 3375 lb., 3742 lb., 3827 lb., 3664 lb., and 3149 lb. If the wagon weighs 1568 lb. and the potatoes are worth 65c. a bushel, how much should the farmer receive?

22. In canning a bushel of peaches, 12 lb. of sugar are used and 20 quart jars of canned fruit are obtained. Find the cost per quart if the peaches cost $2·25, the sugar $1 for 15 lb., the jars $1·05 a dozen, and the labour $1·25.

23. A calf fed on fresh milk will gain 1 lb. in weight for every 1¼ gal. of milk. If milk is 10c. a quart, what will it cost to increase the weight of a calf from 90 to 165 lb.?

24. A contractor undertakes a job that requires the work of 18 men for 15 days. At the end of 5 days, 6 of the men stop work. How many additional men must he employ at the end of the twelfth day, in order that the job may be finished within the contract time of 15 days?

25. A man bought 24 head of cattle. The average weight of the first three was 975 lb.; that of the next four, 1218 lb.; of the next nine, 1124 lb.; and of the remainder 967 lb. What was the average weight of all the 24, and what did they cost at 8½c. a lb.?

26. A farmer sells a grain merchant 33,630 lb. of wheat at $1·20 a bu., 6,817 lb. of oats at 40c. a bu., 16,814 lb. of rye at 80c. a bu., 12,600 lb. of potatoes at 60c. a bu. Make out the bill, inserting names, place, and date. Also receipt the bill.

27. At the beginning of the year 1918 the assets of Messrs. Reed, Henry & Co., were: Cash, $2368·25; Merchandise, $8372; Accounts outstanding, $1364·87. Their liabilities were: Notes, $2385; Accounts unpaid, $1694·50. At the end of the year their assets were: Cash, $4576·28; Merchandise, $9465; Accounts outstanding, $2425·90. Their liabilities were: Notes, $3000; Accounts unpaid, $1267·75. Did the firm gain or lose during the year, and how much?

28. Find the value of:

(a) $39·216 \times ·428 \div ·0642$.

(b) $\dfrac{·5 \text{ of } ·4}{\frac{2}{3} \text{ of } \frac{1}{2}} + \dfrac{\frac{7}{12} \text{ of } 2\frac{3}{5}}{·3 - ·2}$.

(c) $\dfrac{(55·1 - 41·98) \times 4·31}{(6·842 + 3·158) \div ·125}$.

29. Reduce 240 rd. 4 yd. 2 ft. to the decimal of a mile.

30. The average age of the pupils in a school of 300 pupils is 9·75 yr. If 20 new pupils are admitted whose average age is 10·35 yr., what is now the total average age?

31. In 1919 the Champlain Cheese Factory received $18,591·41 for cheese, and $63·43 for interest on deposits in the bank. The factory paid $1828·39 for manufacturing and other expenses. George Crane, one of the shareholders, had, at the end of the year, 71,430 lb. of milk to his credit. If the total amount of milk manufactured was 1,742,323 lb., how much of the factory's profits would Mr. Crane receive?

32. A railway passes through John Miller's farm for a distance of 130 rods, the right of way being 75 ft. wide. At $120 an acre, what is the value of the land taken by the railway?

33. If it costs $150 to fence a square field at 75c. a rod, what will it cost to plough the field at $1·25 an acre?

34. A map is drawn on a scale of one inch to 10 miles, and a township is represented on it by a square whose side is $1\frac{1}{2}$ in. How many acres are there in the township?

35. The average person inhales 30 cubic inches of air at a breath and breathes 18 times in a minute. When one tenth of a given volume of air is breathed, the whole volume is unfit for breathing. In what time would two persons vitiate the air in a bed-room 15 ft. long, 12 ft. wide, and 10 ft. high?

36. A freight car is 36 ft. long, 8 ft. 6 in. wide, and 9 ft. high. How many cubic feet does it contain? If its capacity is 60,000 lb., to what height can it be loaded with wheat?

NOTE.—A bushel = 2218·2 cu. in.

37. A well-ventilated poultry house should furnish 5 sq. ft. of floor space and 30 cu. ft. of air space for each fowl. If the house is 15 ft. wide, what should its other dimensions be to house 60 fowls?

38. If a cubic foot of water contains $6\frac{1}{4}$ gal., find how much sediment will be removed from a 40-acre ploughed field after a 2-in. rainfall, assuming that $\frac{1}{4}$ of the water runs off and that every 300 gal. carries with it 1 lb. of sediment.

39. At $20 a thousand find the total cost of 55 scantlings 18 ft. long, 4 in. wide, and 3 in. thick; 9 scantlings 14 ft. long, 5 in. wide, and 4 in. thick; 8 scantlings 12 ft. long, 6 in. wide, and 5 in. thick.

40. A farmer sent to market 120 lb. of cream containing 25% butter fat. From how many pounds of milk containing 3·75% butter fat was this cream taken?

41. A man sold $\frac{1}{2}$ of a lot of lumber for $\frac{5}{8}$ of what the lot cost. What per cent. was gained on the part sold?

42. The owner of a house was offered $2500 for it. If the offer had been accepted, the loss would have been $16\frac{2}{3}\%$. The house was afterwards sold for $3500. Find the gain or the loss per cent.

43. A dressmaker bought 75 yd. of white lining for $41·25. She sold 60% of it at a gain of $33\frac{1}{3}\%$, and the remainder at 50c. a yard. Find her total gain or loss per cent.

44. A miller pays 50c. a bushel for barley, 70c. a bushel for peas, and 75c. a bushel for corn.   He mixes together 5 bushels of barley, 15 bushels of peas, and 10 bushels of corn, and sells the mixture at $1·50 a cwt. Find his gain per cent.

45. A client gave his lawyer an account for $600, instructing him to collect what he could of it and agreeing to pay him a commission of $2\frac{1}{2}\%$ of the amount collected. If the lawyer's commission amounted to $12, what per cent. of the account did he collect and how much did the client receive?

46. A wholesale firm gives its salesmen a choice of two contracts.   The first is a salary of $2500, with $1500 for expenses and $2\frac{1}{2}\%$ on all sales over $60,000.   The second is a single commission of $5\frac{1}{2}\%$ on all sales.   How much will a salesman gain or lose by taking the second offer if his annual sales are $100,000?

47. A dry-goods merchant sold some cloth at 20% less than the marked price and still made a profit of $8\frac{1}{3}\%$. At what price was the cloth marked if it cost him $240?

48. The following Figure shows the four dials of a gas-meter.   Dial No. 1 records the number of hundred cubic feet of gas; dial No. 2, the number of thousand cubic feet; dial No. 3, the number of ten-thousand cubic feet, etc.

NOTE.—Read the last number which the hand has passed, or that on which it rests.
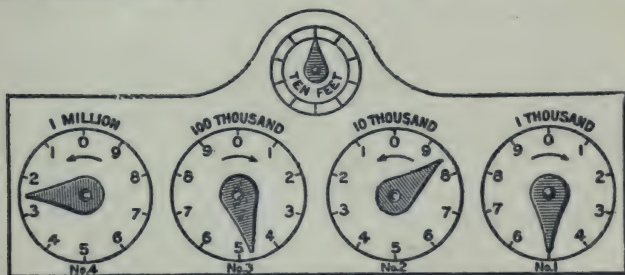
(*a*) How many cubic feet of gas does the gas-meter register?

(b) Find the net cost of the gas at $1·50 a thousand cubic feet with a discount of 16⅔%.

(c) Draw the Figure to show the position of the hands when 86,900 cu. ft. more gas has passed through the meter.

49. The assessed valuation of the real estate of a municipality is $2,350,640, and that of the personal property is $876,500. The year's expenses are: For schools, $10,000; for interest, $2579·96; for roads, $8500; for salaries, $6400; for sinking fund, $8500; and for other expenses, $15,000. The municipality receives $5800 from licenses. What tax must be levied on the dollar to meet the net expenditure?

50. What was the duty on 1250 gallons of spirits invoiced at $6·40 a gallon, there being a specific duty of $1·75 a gallon and an ad valorem duty of 35%? What was the total cost including the duty?

51. A shipment of grain was insured at ⅘% to cover 75% of its value. The premium paid was $33·12. If the grain was worth 92c. a bushel, find the number of bushels.

52. Frank pledged his watch to a pawnbroker for $25. At the end of 40 days he redeemed it for $26·50. What rate of interest did he pay?

53. How much money must be put at simple interest at 8% per annum on Jan. 1st, 1918, so that on March 15th, 1921, there will be $198·85 interest due?

54. In 1919 L. Thompson made deposits in his bank as follows: Jan. 1st, Balance $306; Jan. 20th, $480; Feb. 3rd, $95; Feb. 26th, $375; Mar. 8th, $260; Apr. 10th, $508. His withdrawals were as follows: Jan. 10th, $100; Feb. 10th, $210; Mar. 17th, $125; May 16th, $105. Using the minimum monthly balance, find the total amount of interest at 3% per annum which the bank should add to this account on June 30th, 1919.

55. A farmer has 400 bu. of beans which he can sell at once at $1·20 a bushel. By storing for 6 months at a cost of $15 paid in advance, he can realize $1·30 a bushel. He

takes the latter course. Money being worth 4% per annum, find his gain or loss at the time of sale.

56. On May 10th, 1920, Wm. Turner gave Henry Sills his note for $425 for four months, with interest at the rate of 5% per annum. Write the note and find its value when due.

57. If in 146 days $370 amounts to $376·66, what is the rate of interest per annum?

58. A 3-month note, dated December 10th, 1919, for $1600, with interest at $4\frac{1}{2}$%, was discounted at the bank Jan. 3rd, 1920 at 6%. Find (a) the day of maturity, (b) the value at maturity, (c) the term of discount, (d) the discount, and (e) the proceeds.

59. For the benefit of his son a man invests $4000 at 5% per annum, interest compounded half-yearly. How much will there be to the son's credit in $2\frac{1}{2}$ years?

60. What rate of interest does a man receive on his money who invests it in a $4\frac{1}{2}$% stock at 120?

61. How much money must one invest in a 4% stock at 102, in order to have a net income of $1685 after paying an income tax of 15 mills on the dollar on all over $700?

62. On February 5th, 1920, the value of £1 was quoted at $3·19 in New York.

(a) Find what a New York merchant would pay in New York on that date for a bill for £250 payable in London.

(b) Find what a London merchant would pay in London on that date for a bill for $800 payable in New York.

63. Find to two places of decimals the square root of 3762 and of 68,253.

64. How many rods of fence will inclose an acre in the form of a square? In the form of a circle?

65. The water that falls upon a flat roof 45 ft. × 66 ft. is carried by pipes into a cistern 10 ft. 6 in. in diameter. What will be the depth of the water in the cistern after a rainfall of $3\frac{1}{2}$ in.?

ANSWERS

SECTION I

All exercises in this Section are to be carefully checked by the pupils. Therefore, no answers are given.

SECTION II

16.— **1.** (a) $1426·02, (b) $3931·66, (c) $2195·15, (d) $3113·44.
2. (a) $5014·53, (b) $4257·67, (c) $4533·29, (d) $4034·54.
3. (a) $66611·07, (b) $54502·37, (c) $62145·57.
4. (a) $314733·32, (b) $5552·96, (c) $5750·28.

17.— **1.** (a) $2955·48, (b) $4644·35, (c) $2475·67, (d) $1243·57, (e) $6648·64, (f) $6180·62, (g) $1605·66, (h) $1382·16.
2. (a) $74·63, $58·54, $26·21, $11·15; (b) $753·67, $406·47, $207·08; (c) $4584·97, $2885·49, $342·43.

18.— **1.** (a) 3593546, (b) 3799944, (c) 3727074, (d) 2602926, (e) 4302912, (f) 5169423, (g) 4591302, (h) 2933568, (i) 1503960, (j) 1684202.

19.— **1.** $11·69. **2.** $17·64. **3.** $221·81. **4.** $64. **5.** $23·71. **6.** $377·10. **7.** $40·65. **8.** $50·93.

20.— (a) 175, R. 58; (b) 280, R. 165; (c) 203, R. 295; (d) 237, R. 254; (e) 218, R. 303; (f) 137, R. 258; (g) 181, R. 55; (h) 73, R. 449; (i) 67, R. 69.

21.— **1.** $35·24. **2.** $7·85. **3.** $149·50. **4.** $136·50. **5.** 65 lb. **6.** 58 bu. 18 lb. **7.** $45. **8.** $2·66. **9.** 102 bu. 30 lb. **10.** $131·25. **11.** 31 T. **12.** $115·20. **13.** $1423.

EXERCISE

22.— 1. (a) $2 \times 2 \times 3 \times 3 \times 3 \times 3$, (b) $2 \times 2 \times 2 \times 2 \times 3 \times 7$, (c) $2 \times 3 \times 5 \times 13$, (d) $2 \times 2 \times 2 \times 7 \times 7$, (e) $2 \times 2 \times 3 \times 5 \times 7$, (f) $3 \times 3 \times 5 \times 11$, (g) $5 \times 11 \times 11$, (h) $2 \times 2 \times 2 \times 3 \times 5 \times 7$, (i) $2 \times 3 \times 3 \times 7 \times 7$, (j) $2 \times 3 \times 3 \times 109$, (k) $3 \times 5 \times 17 \times 17$, (l) $2 \times 5 \times 7 \times 7 \times 11$.

23.— 1. 87. **2.** 137. **3.** 129. **4.** 87. **5.** 137. **6.** 223. **7.** 834. **8.** 368. **9.** 341. **10.** 124. **11.** 923. **12.** 836.

24.— 1. (a) 57c. (b) \$5·10, (c) \$2·13, (d) \$7·70, (e) \$66·45.

25.— 1. Bal. \$32·55. **2.** Bal. \$7·75. **3.** Bal. \$28·25. **4.** Bal. 0.

26.— No answers are necessary.

27.— 3. 13. **4.** 165 lb., 55 lb. **5.** 324 lb., 81 lb. **6.** \$4·80, 80c. **7.** \$22·30, \$4·46. **8.** Av. \$2·58. **9.** 98c., 14c. **10.** 8700 lb., 2175 lb.

28.— 7. \$10·08. **8.** \$8·40. **9.** 408.

29.— 1. 24d., 60d., 240d., 240d. **2.** 2s., 3s., 2s. 6d., 4s. 2d., 8s. 4d. **3.** 9072d., 147d., 189d. **4.** 20s., 100s., 240s., 500s. **5.** 106s., 247s., 502s. **6.** 2943d. **7.** £4 2s. 1d. **8.** £3 18s. 6d.

30.— 1. 5280, 12320, 7260, 9240. **2.** $\frac{1}{2}$, $\frac{1}{4}$. **3.** 10. **4.** \$16170. **5.** \$3300. **6.** 15 sec. **7.** 5280. **8.** 3 mi., 180 mi. **9.** \$462. **10.** 704.

31.— 7. \$1·75. **8.** \$63. **9.** \$5·20. **10.** \$553. **11.** \$24. **12.** \$108.

32.— 1. \$114·80. **2.** \$83·70. **3.** \$73·44. **4.** \$189. **5.** \$147·50. **6.** \$498·75. **7.** \$459. **8.** \$343·44. **9.** \$301·84. **10.** \$168.

EXERCISE

33.— 1. 5 hr.   2. 7 hr. 30 min.   3. 7 hr. 15 min.
4. 18 hr. 15 min.   5. (a) 16 dy.,   (b) 61 dy.,
(c) 101 dy.,   (d) 259 dy.,   (e) 356 dy.   7. $45.
8. $53·25.   9. (a) $36·72, $34, $37·40, $36·38,
$21·08, $28·56, $40·80, $36·72;   (b) $271·66;
(c) $19·72.

34.— 1. $7·54.   2. $2·20.   3. 70 yr.   4. $4·32.
5. $21·60.   6. $6·40.   7. $17·28.   8. 2 rm.
9. $6·60.   10. $7·50.

35.— 1. (a) $1\frac{1}{2}$, (b) 2, (c) 1, (d) $\frac{5}{7}$, (e) $\frac{7}{8}$, (f) $\frac{9}{10}$, (g) $\frac{11}{13}$, (h) $\frac{7}{10}$, (i) $\frac{41}{50}$.
2. (a) $\frac{3}{4}$, (b) $1\frac{1}{4}$, (c) $\frac{1}{2}$, (d) $\frac{5}{8}$, (e) $\frac{1}{2}$, (f) 1, (g) $1\frac{1}{4}$, (h) $1\frac{8}{10}$, (i) $\frac{17}{20}$.
3. (a) $1\frac{7}{12}$, (b) $1\frac{7}{15}$, (c) $1\frac{1}{10}$, (d) $1\frac{19}{30}$, (e) $\frac{11}{14}$, (f) $1\frac{6}{35}$, (g) $1\frac{7}{24}$, (h) $1\frac{5}{63}$, (i) $1\frac{19}{55}$.
4. (a) $1\frac{1}{30}$, (b) $\frac{3}{4}$, (c) $1\frac{19}{48}$, (d) $1\frac{7}{12}$, (e) $1\frac{2}{5}$, (f) $1\frac{19}{24}$, (g) $2\frac{1}{4}$, (h) $1\frac{27}{40}$, (i) $1\frac{53}{84}$, (j) $1\frac{1}{15}$, (k) $1\frac{6}{7}$, (l) $1\frac{13}{20}$.
5. $2\frac{1}{3}$ 6. $1. 7. $1\frac{3}{8}$ lb. 8. $1\frac{11}{20}$. 9. $\frac{3}{4}$ ac. 10. $\frac{11}{12}$ gal.
11. $2\frac{8}{20}$.

36.— 2. (a) $7\frac{5}{8}$, (b) $10\frac{29}{63}$, (c) $8\frac{11}{18}$, (d) $10\frac{1}{2}$, (e) $10\frac{13}{14}$,
(f) $9\frac{31}{40}$. 3. $14\frac{7}{13}$. 4. $92\frac{1}{4}$. 5. 84 yd. 6. $93\frac{1}{4}$c.
7. $23\frac{7}{13}$ hr. 8. $10\frac{5}{12}$.

37.— 1. (a) $\frac{2}{8}$, (b) $\frac{5}{9}$, (c) $\frac{1}{4}$, (d) $\frac{3}{10}$, (e) $\frac{1}{4}$, (f) $\frac{1}{4}$, (g) $\frac{1}{8}$,
(h) $\frac{5}{12}$, (i) $\frac{2}{15}$, (j) $\frac{3}{25}$, (k) $\frac{5}{18}$, (l) $\frac{9}{10}$, (m) $\frac{5}{14}$,
(n) $\frac{19}{63}$, (o) $\frac{11}{88}$, (p) $\frac{7}{80}$, (q) $\frac{5}{24}$, (r) $\frac{18}{35}$, (s) $\frac{25}{132}$,
(t) $\frac{1}{80}$.
2. $\frac{41}{84}$. 3. $\$\frac{5}{24}$. 4. $\frac{9}{40}$ ac. 5. $1\frac{1}{11}$.

38.— 1. (a) $524972·66, (b) $269875·14, (c) $1535778·16.
2. (a) $119166·07, (b) $554451·67, (c) $488856·45.
3. (a) 40525524, (b) 54637996, (c) 315182725.
4. (a) 1738, R. 265; (b) 3185, R. 644; (c) 81, R. 1486.

38.— **5.** (a) 33002, (b) 17075, (c) 198492, (d) 47362.
  6. (a) 6210, (b) 8064, (c) 18032, (d) 26013,
  7. \$3·69. **9.** 61¼c. **10.** 3034 lb. **11.** \$76·05·
  12. \$58·64. **13.** 786d. **14.** (a) \$58·90, (b) \$58·80,
  (c) \$53·10, (d) \$97, (e) \$300, (f) \$21, (g) \$133·50.
  15. 216, \$540. **16.** 72 dy. **17.** 8 gal. **18.** \$5·25.
  19. 15 bbl. **20.** \$340. **21.** \$11·25. **22.** 775000.
  23. \$1·82. **24.** \$77. **25.** 26 lb. **26.** \$190.
  27. 103 bu. **28.** \$28·86, Bal. **29.** $\frac{7}{2}$, $\frac{21}{4}$, $\frac{20}{3}$, $\frac{61}{8}$, $\frac{42}{5}$.
  30. $\frac{8}{13}$, $\frac{12}{11}$, $\frac{20}{24}$, $\frac{42}{48}$. **31.** 15$\frac{19}{80}$. **32.** (a) $\frac{13}{18}$, (b) $\frac{25}{36}$,
  (c) $\frac{17}{84}$.

SECTION III

39.— No answers necessary.

40.— **1.** (a) \$10·17, (b) \$11·72, (c) \$15·22, (d) \$18·17,
  (e) \$8·12. **3.** (a) \$487·34, (b) \$771·39.

41.— **1.** \$28·25. **2.** \$29. **3.** \$86. **4.** \$28·08. **5.** \$9·90.

42.— **1.** 15. **2.** 36. **3.** 12. **4.** 17. **5.** 17. **6.** 13. **7.** 12.
  8. 91. **9.** 29. **10.** 512. **11.** 31. **12.** 16.

43.— **1.** 590, R. 19; 564, R. 1; 517, R. 1; 496, R. 9.
  2. 1945, R. 34; 1751, R. 14; 1667, R. 40; 1592, R. 6;
  1459, R. 22. **3.** 1714, R. 1; 1402, R. 13; 826, R. 23;
  642, R. 55; 550, R. 79. **4.** 672, R. 94. **5.** 933, R. 35

44.— **1.** (a) 3, (b) 2, (c) 4, (d) 9, (e) 5, (f) 2$\frac{2}{3}$, (g) 1$\frac{1}{7}$,
  (h) 1$\frac{1}{3}$, (i) 1$\frac{1}{3}$, (j) 2$\frac{2}{3}$, (k) 3$\frac{1}{4}$, (l) 3, (m) 3, (n) 5,
  (o) 2, (p) $\frac{2}{5}$, (q) $\frac{5}{25}$, (r) 3$\frac{3}{55}$, (s) 45, (t) 12. **2.** 100 bu.
  3. 21 bu. **4.** 180. **5.** 27. **6.** 5.

45.— **1.** 720. **2.** 1344. **3.** 756 **4.** 1650. **5.** 1050.
  6. 16800 **7.** 720. **8.** 2002. **9.** 144. **10.** 780.
  11. 2340. **12.** 960.

EXERCISE

46.— 1. $\frac{88}{5}$. 2. $\frac{63}{4}$. 3. $\frac{419}{14}$. 4. $\frac{880}{13}$. 5. $\frac{479}{8}$. 6. $\frac{858}{8}$.
7. $\frac{398}{15}$. 8. $\frac{505}{11}$. 9. $\frac{801}{14}$. 10. $\frac{495}{8}$. 11. $\frac{2751}{3}$.
12. $\frac{1007}{1}$. 13. $\frac{1423}{11}$. 14. $\frac{2455}{44}$. 15. $\frac{1429}{6}$.
16. $\frac{6185}{16}$. 17. $\frac{674}{9}$. 18. $\frac{1528}{11}$

47.— 1. $18\frac{6}{25}$. 2. $28\frac{1}{14}$. 3. $26\frac{5}{6}$. 4. $19\frac{6}{41}$. 5. $80\frac{3}{8}$.
6. $21\frac{44}{45}$. 7. $23\frac{29}{37}$. 8. $18\frac{7}{6}$. 9. $10\frac{1}{4}$. 10. $90\frac{1}{4}$.
11. $100\frac{34}{76}$. 12. $292\frac{7}{20}$.

48.— 1. $\frac{2}{3}$. 2. $\frac{5}{12}$. 3. $\frac{1}{3}$. 4. $\frac{11}{37}$. 5. $\frac{7}{8}$. 6. $\frac{3}{8}$. 7. $\frac{3}{4}$.
8. $\frac{9}{16}$. 9. $\frac{5}{12}$. 10. $\frac{4}{9}$. 11. $\frac{5}{6}$. 12. $\frac{2}{3}$. 13. $\frac{15}{25}$.
14. $\frac{9}{11}$. 15. $\frac{3}{10}$. 16. $\frac{4}{9}$. 17. $\frac{17}{20}$. 18. $1\frac{1}{2}$.
19. $\frac{3}{4}$. 20. $1\frac{2}{3}$. 21. $\frac{9}{11}$. 22. $\frac{2}{3}$. 23. $\frac{11}{12}$. 24. $1\frac{3}{4}$.
25. $\frac{7}{9}$. 26. $\frac{1}{3}$. 27. $\frac{5}{13}$. 28. $\frac{6}{7}$.

49.— 1. $\frac{4}{8}$, $\frac{2}{8}$, $\frac{1}{8}$.   2. $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$.   3. $\frac{6}{15}$, $\frac{3}{15}$, $\frac{2}{15}$.
4. $\frac{5}{10}$, $\frac{2}{10}$, $\frac{1}{10}$.   5. $\frac{8}{24}$, $\frac{3}{24}$, $\frac{2}{24}$.   6. $\frac{8}{24}$, $\frac{18}{24}$, $\frac{9}{24}$.
7. $\frac{15}{30}$, $\frac{20}{30}$, $\frac{24}{30}$.   8. $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.   9. $\frac{35}{40}$, $\frac{12}{40}$, $\frac{20}{40}$.
10. $\frac{6}{12}$, $\frac{10}{12}$, $\frac{9}{12}$.   11. $\frac{18}{42}$, $\frac{15}{42}$, $\frac{22}{42}$.   12. $\frac{40}{72}$, $\frac{28}{72}$, $\frac{33}{72}$.
13. $\frac{15}{100}$, $\frac{16}{100}$, $\frac{18}{100}$.   14. $\frac{40}{100}$, $\frac{35}{100}$, $\frac{41}{100}$.   15. $\frac{75}{100}$,
$\frac{90}{100}$, $\frac{42}{100}$, $\frac{81}{100}$.   16. $\frac{25}{50}$, $\frac{10}{50}$, $\frac{5}{50}$, $\frac{2}{50}$.

50.— 1. 1. 2. $1\frac{1}{3}$. 3. 1. 4. $\frac{1}{4}$. 5. $\frac{4}{5}$. 6. $\frac{1}{5}$. 7. $1\frac{1}{12}$.
8. 1. 9. $\frac{4}{5}$. 10. $2\frac{1}{10}$. 11. $\frac{5}{6}$. 12. $1\frac{1}{5}$. 13. $\frac{3}{4}$.
14. $\frac{1}{4}$. 15. $1\frac{1}{16}$. 16. $\frac{5}{16}$. 17. $\frac{13}{32}$. 18. $1\frac{1}{2}$.
19. $\frac{4}{15}$. 20. $2\frac{1}{6}$. 21. $1\frac{13}{15}$. 22. $\frac{1}{16}$. 23. $1\frac{5}{6}$.
24. $2\frac{1}{6}$. 25. $\frac{5}{6}$. 26. $\frac{7}{16}$. 27. $1\frac{5}{12}$. 28. $\frac{1}{80}$.
29. $1\frac{3}{20}$. 30. $2\frac{8}{10}$. 31. $\frac{7}{20}$. 32. $\frac{13}{14}$. 33. $1\frac{7}{24}$.
34. $1\frac{18}{48}$. 35. $\frac{1}{12}$. 36. $1\frac{8}{40}$. 37. $\frac{1}{24}$. 38. $1\frac{2}{15}$.
39. $\frac{11}{15}$. 40. $\frac{13}{20}$. 41. $1\frac{7}{40}$. 42. $1\frac{3}{8}$ bbl. 43. $\frac{4}{5}$, $\frac{1}{5}$.
44. $1\frac{3}{16}$ mi. 45. $\$\frac{1}{20}$.

51.— 1. (a) $12\frac{5}{6}$, (b) $2\frac{2}{5}$, (c) $6\frac{1}{6}$, (d) $3\frac{1}{8}$, (e) $5\frac{5}{12}$, (f) $2\frac{4}{5}$,
(g) $8\frac{1}{2}$, (h) $3\frac{1}{8}$, (i) $\frac{5}{6}$, (j) $2\frac{9}{10}$, (k) $7\frac{11}{12}$, (l) $4\frac{13}{24}$,
(m) $15\frac{9}{40}$, (n) $6\frac{41}{70}$, (o) $6\frac{1}{4}$.   2. $16\frac{5}{12}$.   3. $19\frac{1}{24}$ ft.
4. $5\frac{11}{12}$c.   5. $15\frac{13}{60}$ hr.

EXERCISE

52.— 1. 380. **2.** \$10·92. **3.** \$187·50. **4.** \$300.
5. $18\frac{3}{4}$ mi. **6.** 100.

53.— 1. 20 pt. **2.** 300 min. **3.** 1200 lb. **4.** 1320 yd.
5. \$740. **6.** \$20. **7.** $\frac{2}{15}$. **8.** $19\frac{7}{8}$ mi., $19\frac{7}{8}$ mi., $6\frac{7}{8}$ mi.

54.— 1. (a) $\frac{4}{15}$, (b) $\frac{5}{104}$, (c) $\frac{4}{9}$, (d) $\frac{5}{22}$, (e) $\frac{2}{5}$, (f) $\frac{1}{60}$,
(g) $\frac{1}{100}$, (h) $\frac{1}{51}$, (i) $\frac{1}{100}$, (j) $\frac{11}{26}$, (k) $\frac{3}{60}$, (l) $\frac{6}{25}$,
(m) $1\frac{1}{7}$, (n) $1\frac{49}{44}$, (o) $1\frac{3}{5}$, (p) $\frac{14}{88}$, (q) $\frac{7}{10}$, (r) $\frac{3}{4}$,
(s) $1\frac{9}{16}$, (t) $\frac{3}{25}$.
2. $\frac{23}{240}$. **3.** $\frac{1}{45}$. **4.** 96 ac.
5. $7\frac{5}{16}$. **6.** 35c.

55.— 1. (a) $\frac{2}{105}$, (b) $\frac{3}{8}$, (c) $\frac{19}{27}$, (d) $\frac{23}{40}$, (e) $\frac{4}{15}$, (f) $1\frac{1}{2}$,
(g) 6, (h) $6\frac{2}{3}$, (i) 14, (j) 30, (k) 39, (l) $\frac{1}{8}$, (m) $9\frac{13}{18}$,
(n) $31\frac{1}{3}$, (o) 1, (p) $103\frac{59}{125}$, (q) $\frac{3}{10}$, (r) $9\frac{4}{5}$, (s) $3\frac{5}{13}$,
(t) $3\frac{3}{32}$.
2. $12\frac{1}{2}$. **3.** $26\frac{1}{4}$. **4.** $185\frac{5}{8}$ mi.
5. (a) \$21·35, (b) \$9·46$\frac{7}{8}$, (c) \$10·10$\frac{5}{8}$, (d) \$5·04$\frac{1}{3}$.

56.— 1. 1. **2.** 2. **3.** $\frac{2}{3}$. **4.** $\frac{2}{3}$. **5.** $1\frac{1}{3}$. **6.** $1\frac{1}{3}$. **7.** $\frac{3}{4}$.
8. $\frac{3}{4}$. **9.** $\frac{11}{21}$. **10.** $1\frac{7}{20}$. **11.** $\frac{14}{27}$. **12.** $2\frac{17}{32}$.
13. $8\frac{1}{3}$. **14.** $1\frac{3}{7}$. **15.** $\frac{3}{4}$. **16.** $2\frac{1}{2}$. **17.** $1\frac{4}{7}$.
18. $\frac{11}{15}$. **19.** $\frac{49}{104}$. **20.** 10. **21.** $10\frac{1}{2}$. **22.** 16.
23. $26\frac{2}{3}$. **24.** 18. **25.** 4. **26.** 10. **27.** $1\frac{1}{5}$.
28. $1\frac{1}{15}$. **29.** $\frac{1}{3}$. **30.** $29\frac{4}{25}$. **31.** 19. **32.** \$50.
33. $3\frac{1}{2}$ hr. **34.** 14.

57.— 1. $\frac{7}{18}$. **2.** $\frac{1}{6}$. **3.** $\frac{1}{4}$. **4.** $\frac{7}{20}$. **5.** $\frac{11}{21}$. **6.** $\frac{18}{23}$.
7. (a) $\frac{89}{90}$, (b) $\frac{7}{72}$, (c) $\frac{5}{18}$, (d) $\frac{10}{7}$, (e) $\frac{14}{9}$, (f) $\frac{3}{5}$ (g) $\frac{3}{4}$.

58.— 1. $\frac{4}{5}$. **2.** $\frac{3}{7}$. **3.** $\frac{1}{8}$. **4.** 2. **5.** $2\frac{1}{2}$. **6.** $6\frac{1}{18}$. **7.** $\frac{5}{6}$.
8. $1\frac{9}{16}$. **9.** $\frac{49}{77}$. **10.** $\frac{5}{6}$. **11.** 2. **12.** 5. **13.** $1\frac{21}{44}$.
14. $2\frac{3}{4}$.

59.— 1. (a) $\frac{3}{7}$, (b) $\frac{3}{10}$, (c) $\frac{9}{7}$, (d) $\frac{3}{10}$, (e) $\frac{7}{12}$, (f) $\frac{13}{27}$, (g) $\frac{5}{8}$.
2. (a) $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{50}{60}$, $\frac{35}{60}$; (b) $\frac{30}{84}$, $\frac{28}{84}$, $\frac{56}{84}$, $\frac{70}{84}$, $\frac{54}{84}$, $\frac{22}{84}$;
(c) $\frac{50}{100}$, $\frac{20}{100}$, $\frac{80}{100}$, $\frac{14}{100}$, $\frac{82}{100}$. **3.** (a) $\frac{24}{88}$, $\frac{20}{88}$, $\frac{27}{88}$,
$\frac{44}{88}$; (b) $\frac{18}{42}$, $\frac{22}{42}$, $\frac{27}{42}$, $\frac{28}{42}$; (c) $\frac{20}{60}$, $\frac{465}{60}$, $\frac{252}{60}$, $\frac{9}{60}$, $\frac{148}{60}$.

EXERCISE

59.— 4. (a) $1\frac{5}{8}$, (b) $2\frac{18}{65}$, (c) $11\frac{91}{200}$.   **5.** (a) $\frac{1}{8}$, (b) $\frac{1}{56}$, (c) $3\frac{8}{10}$, (d) $1\frac{7}{18}$.   **6.** (a) $\frac{189}{512}$, (b) $1\frac{5}{8}$, (c) $\frac{9}{175}$.
7. (a) $\frac{5}{21}$, (b) $\frac{5}{18}$, (c) $2\frac{4}{18}$, (d) $\frac{1}{4}$.   **8.** (a) $\frac{1}{3}$, (b) $\frac{3}{14}$, (c) $\frac{29}{2}$, (d) 42, (e) 105, (f) 375, (g) $84\frac{1}{2}$, (h) $123\frac{1}{2}$, (i) $164\frac{8}{25}$.   **9.** (a) $\frac{2}{15}$, (b) $\frac{1}{4}$, (c) $\frac{9}{40}$, (d) 16, (e) $22\frac{1}{2}$, (f) 20, (g) $1\frac{7}{8}$, (h) $\frac{5}{85}$, (i) $\frac{217}{288}$.   **10.** 70.   **11.** 20 mi.
12. 130 lb.   **13.** 50 ft.   **14.** $58\frac{1}{3}$.   **15.** 250.
16. 15.   **17.** $11·24.   **18.** $59·01.   **19.** $78·64.

60.— 1. 640, 960, 800, 1200, 1390.   **2.** 2, $\frac{1}{2}$, $\frac{1}{8}$, 4, 7.
3. 4 mi. 120 rd.; 4 mi. 280 rd.; 5 mi. 160 rd.
4. 20.   **5.** 400.   **6.** $70.   **7.** 11, 33, 77, $60\frac{1}{2}$,
$82\frac{1}{2}$.   **8.** 4, 6, 10, 16.   **9.** 1 rd. $2\frac{1}{2}$ yd.; 2 rd. 1 yd.;
3 rd. $2\frac{1}{2}$ yd.; 4 rd. 5 yd.   **10.** $10780.
11. $31·50.   **12.** $3892\frac{1}{2}$.

61.— 1. 5 in.   **2.** $2\frac{1}{2}$ in.   **3.** $2\frac{1}{2}$ in.   **4.** 2 in. × $1\frac{1}{2}$ in.,
7 in., 56 ft.   **5.** (a) 3 in. × 2 in., 80 ft.; (b) $1\frac{1}{2}$
in. × $2\frac{1}{4}$ in., 60 ft.; (c) $2\frac{1}{2}$ in. × $1\frac{3}{4}$ in., 68 ft.
6. (a) 2 in. × $1\frac{1}{2}$ in., 112 yd.; (b) 3 in. × $2\frac{1}{4}$ in.,
168 yd.; (c) $3\frac{1}{2}$ in. × $2\frac{5}{8}$ in., 196 yd.   **7.** (a) 2 in.
× 2 in., 160 rd.; (b) 4 in. × 1 in., 200 rd.; (c)
3 in. × $1\frac{1}{2}$ in., 180 rd.   **8.** $28·05.   **9.** $123·48.
10. 420.   **11.** 8 ft.   **12.** $280·50.   **13.** 264.
14. 133.   **15.** $1\frac{5}{8}$ mi.

62.— A—4. 3 sq. ft. 18 sq. in.; 2 sq. ft. 94 sq. in.;
6 sq. ft. 11 sq. in.; 6 sq. ft. 36 sq. in.   **5.** 32.
6. $90\frac{5}{18}$.   **7.** 22176.
B—4. 648.   **5.** 88.   **6.** 16.
C—2. $60\frac{1}{2}$, $90\frac{3}{4}$, 121, 133, $206\frac{1}{2}$.   **3.** 48400.
4. (a) 2420, (b) $2722\frac{1}{2}$, (c) 3267.
D—4. (a) 30, (b) 15, (c) 50, (d) 640, (e) 9, (f) $21\frac{3}{8}$.
E—2. 640, 640, 1280, 320, 480, 1440, 2400, 2880.

63.— 1. $4\frac{3}{4}$, 9, 12, $24\frac{8}{10}$.   **3.** $14040.   **4.** $450.
5. $1366·20.   **6.** $2155·68.   **7.** $207·90.

EXERCISE

63.— **8.** 16 rd.   **9.** $17·28.   **10.** 54.   **11.** $16·67.
12. 18 rd., $2\frac{1}{10}$.

64.— **1.** 3456, 5184, 6912, 3492, 8712.   **2.** 6.   **3.** 18.
4. 19 cu. ft. 822 cu. in.

65.— **1.** 54, 99, 151, 200.   **2.** 224.   **3.** 92583.   **4.** 630.
5. 630, $472·50.

66.— **1.** $4\frac{1}{2}$.   **2.** 12.   **3.** 12.   **4.** $40\frac{1}{2}$.   **5.** 144.   **6.** $26\frac{1}{4}$.
7. $70\frac{7}{8}$.   **8.** $2160, $393·75, $1063·$12\frac{1}{2}$.

67.— **1.** 10000.   **2.** 24, 20.   **3.** $150.   **4.** $126.   **5.** 36.
6. 800.   **7.** 224.   **9.** $126.   **10.** $392.   **11.** 1200.
12. $\frac{55}{64}$ ft.

68.— **1.** (a) $12\frac{1}{2}$, (b) $17\frac{1}{2}$, (c) 16, (d) 90, (e) $14\frac{7}{12}$, (f) $11\frac{13}{18}$,
(g) $11\frac{1}{4}$, (h) $9\frac{1}{8}$, (i) $18\frac{2}{3}$, (j) $10\frac{15}{8}$.   **2.** 360 bd. ft.
3. $113·40.   **4.** $45.   **5.** $32·50.   **6.** $117·$83\frac{1}{3}$.

69.— No answers are necessary.

70.— **6.** $\frac{9}{40}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$, 1.   **7.** $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{8}{10}$, $\frac{2}{5}$,
$\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, 1.   **8.** $\frac{1}{3}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{19}{20}$.

71.— **5.** (a) $2\frac{2}{3}$, (b) $\frac{115}{243}$.   **6.** (a) $5 \times 7 \times 11 \times 11$,
(b) $2 \times 3 \times 3 \times 7 \times 17$, (c) $5 \times 3 \times 7 \times 7 \times 7$,
(d) $11 \times 11 \times 11$.   **7.** (a) 136, (b) 99.
8. (a) 2080, (b) 1330.   **9.** (a) $\frac{4}{7}$, (b) $\frac{2}{5}$, (c) $\frac{4}{9}$,
(d) $\frac{4}{7}$.   **10.** (a) $\frac{1}{4}$, (b) $1\frac{49}{60}$, (c) $\frac{23}{24}$, (d) $28\frac{1}{24}$, (e) $14\frac{17}{20}$,
(f) $\frac{3}{10}$, (g) $\frac{61}{98}$, (h) $\frac{16}{125}$, (i) $75\frac{3}{5}$, (j) $\frac{7}{13}$.   **11.** (a) 19
sq. ft.,   (b) $101\frac{1}{2}$,   (c) $139\frac{1}{3}$.   **12.** 24, 20, $15\frac{1}{2}$.
13. (a) 567, (b) 1400, $10\frac{15}{16}$, (c) $3867\frac{3}{16}$.   **14.** (a) 160,
(b) 270, (c) 1120.   **15.** (a) $140\frac{5}{8}$, (b) $7871\frac{3}{32}$.
16. (a) 13,   (b) $11\frac{7}{10}$.   **17.** (a) $25\frac{1}{5}$,   (b) $21\frac{3}{5}$.
18. (a) 288,   (b) 527,   (c) $700\frac{3}{20}$.   **19.** $7\frac{5}{16}$.
20. (a) $13\frac{1}{2}$,   (b) 30,   (c) 35.   **21.** (a) $\frac{19}{800}$,
(b) $\frac{3}{8}$, (c) $\frac{1}{2}$, (d) $\frac{1}{3}$, (e) $\frac{5}{8}$, (f) $\frac{4}{5}$, (g) $\frac{7}{8}$, (h) $\frac{29}{48}$.

EXERCISE

71.— 22. (a) 75, (b) 87½, (c) 37½, (d) 62½, (e) 85⅚,
(f) 83⅓, (g) 58½, (h) 66⅔, (i) 50, (j) 60.
23. (a) $90, (b) $120, (c) $191, (d) $605, (e) $567.
24. $227·50. **25.** $3675. **26.** $53·51¼.
27. $21·09. **28.** $6·50. **29.** $48·48 Cr. **30.** $9·85.
31. $107·01. **32.** $12·58$\frac{11}{18}$. **33.** 130.
34. 12$\frac{21}{32}$, $5⅝. **35.** 64. **36.** $225. **37.** $2090.
38 (a) $54, (b) $88·59⅜, (c) $39·60, (d) $393·75,
(e) 819, (f) 945.

SECTION IV

72.— 1. 3·59. **2.** 711. **3.** 179·47. **4.** 216·692.
5. 15476·306. **6.** $140·83. **7.** $788·804.
8. 45·53783, 997·154, 9·99577, 448·757.
9. 31·9773. **10.** $83·3165. **11.** 995·5 mi.
12. 337·63. **13.** 285·675 mi. **14.** 29·2.
15. ·098.

73.— 1. (a) 29·6, (b) 41·31, (c) 5269·081, (d) 2617·2,
(e) 63·08, (f) 48·042, (g) 3228·396, (h) 2·5935,
(i) ·0174, (j) 1725·0748, (k) 257·004, (l) 25243·75,
(m) 5·02866, (n) 47·892, (o) ·01944, (p) ·1148488.
2. 43·2156, 432·156, 4321·56. **3.** 5463·7,
13784·6, 297836. **4.** 295·275. **5.** 7. **6.** 2772.
7. 696. **8.** 5. **9.** 57·12. **10.** $540. **11.** 39·132.
12. $924·60. **13.** $1000. **14.** $131·3955.
15. $66·8625. **16.** 468·8 ft.

74.— 1. 159·792 mi. **2.** 250·2835, 27·8092 tons.
3. ·481 lb. **4.** 1·077 lb. **5.** $12·71.
6. $103·17. **7.** $57·22. **8.** $31·52.
9. $120·98.

75.— 1. (a) $50, (b) $70, (c) $31·50, (d) $18, (e) $7·35,
(f) $1·87½, (g) $1·40, (h) $2·02½.

75.— **2.** (a) \$15, \$90; (b) \$32, \$64; (c) \$28·12½, \$253·12½; (d) \$13·25, \$145·75.

76.— **1.** (a) 60, (b) 62½, (c) 80, (d) 62½, (e) 25, (f) $71\frac{7}{8}$, (g) 15, (h) 70, (i) $9\frac{1}{11}$, (j) 20, (k) 16⅔, (l) 16⅔. **2.** (a) 33⅓, (b) 20, (c) 33⅓, (d) 20, (e) 20, (f) 12½. **3.** \$4·96⅞, \$5·94⅜, \$7·84¾. **4.** \$854·40. **5.** 22⅔%. **6.** 10½c.

77.— **1.** (a) 60, (b) 66⅔, (c) $71\frac{3}{7}$, (d) 180, (e) 150, (f) $111\frac{1}{5}$ (g) 57½, (h) 133⅓. **2.** $10\frac{4}{5}$. **3.** 50. **4.** \$1125. **5.** \$14·62½. **6.** \$9·69.

78.— **1.** 58·45. **2.** \$102·75. **3.** \$148·90. **4.** \$242·35. **5.** 479·85. **6.** 7180·07.

79.— No answers are necessary.

80.— **1.** \$61·88. **2.** \$69·70. **3.** \$132·52. **4.** \$116·42.

81.— **1.** 2 D.S.C., 10 T.S. **2.** \$35. **3.** \$25, \$50. **4.** 8 D.S.C., \$40, 12 T.S.

82.— **1.** 8⅓. **2.** \$132·22. **3.** \$2·78. **4.** \$359. **5.** \$138·75. **7.** 199·1 lb., 234·235 lb. **8.** \$4·68¾. **9.** 57 lb. **10.** \$691·60. **11.** \$44·67. **14.** 300, 400, 280, 10. **15.** 280, 440, 240. **16.** 66⅔ lb. **17.** 50c., 25c., 20c. **18.** 60 lb., 80 lb., 0. **19.** \$116·25, \$771·25.

83.— **1.** \$3·39½. **2.** \$260·87½. **3.** \$391·97. **4.** \$3·98¼. **5.** \$245·32. **6.** \$3·9852. **7.** \$7·22. **8.** \$433·50. **9.** \$43·63. **10.** \$1·76½. **11.** 22c. **12.** 4668⅔ calories. **13.** $16\frac{11}{20}$c.

84.— **1.** \$328·32. **2.** \$193·05. **3.** \$408·33⅓. **4.** \$96·25. **5.** \$9975. **6.** \$13082. **7.** \$2433·60.

85.— **1.** 12. **2.** 15. **3.** \$1·75, \$3·12.

86.— **1.** (a) 72, (b) 66, (c) 205, (d) 168, (e) 219, (f) 213. **2.** \$217·25. **3.** \$327·12. **4.** \$384·90.

EXERCISE

87.— **1.** $27·81. **2.** $95·20. **3.** $90·30. **4.** $103·19.
5. $65·92.

88.— **1.** (a) 288, (b) 420, (c) 225⅝. **2.** (a) 208⅛, (b) 165²⅘,
(c) 241¹¹⁄₂₄. **3.** $80. **4.** $148·75.

89.— **1.** $67·50. **2.** $56·66. **3.** 7, 6, 42, $105.
5. $33, $409·20. **6.** 13½ ft. × 10½ ft.; $118·12;
$11·68.

90.— **1.** 22. **2.** $116·74. **3.** $11·08. **4.** 35¹⁄₁₈.
5. $60·19, $375·38, $627, $1062·57.

91.— **1.** 7. **2.** 4. **3.** 5. **4.** 3. **5.** 10. **6.** 16. **7.** 9.
8. 6. **9.** 9. **10.** 6. **11.** 7. **12;** 4.

92.— **5.** (a) 3·86543, (b) $·2592, (c) 8·712, (d) $3·39.
7. 480·973 mi. **8.** 27720 yd. **9.** $334·09.
10. $526·4615. **11.** ¼, ⅓, ⅓, ⅜, ⅝, ⅞, ⅚, ⅔. **12.** 33⅓%.
13. 12½%. **14.** 1000. **15.** 15, 25, 38. **16.** 1 D.S.C.,
7 T.S.,; 1 D.S.C., 13 T.S.; 2 D.S.C., 6 T.S.;
3 D.S.C., 9 T.S. **17.** $100, $60, $75. **18.** $150·65.
20. $74·86. **21.** $43·75. **22.** (a) $47·69,
(b) $45·85. **23.** $325·30. **24.** $4·76¼. **25.** $12.
26. 8⅛. **27.** $19140. **28.** (a) $28·50, (b) $382·63⅜.
29. 13. **30.** $48·55. **31.** $20·52½.
32. $231·71. **33.** $270·10. **34.** $369·68.
35. $36·75.

SECTION V

93.— **1.** 2, 7, 11, 14, 22, 77, 154; 2, 3, 4, 6, 8, 9, 12, 16,
18, 24, 27, 36, 48, 54, 72, 108, 144, 216; 23, 29;
3, 5, 7, 9, 15, 21, 27, 35, 45, 63, 105, 135, 189, 315.
2. 2, 2, 3, 3, 19; 29, 29; 11, 11, 11; 2, 2, 2, 3, 3,
5, 7; 3, 5, 7, 11, 17; 2, 2, 3, 3, 13, 103. **3.** Prime
269, 1367. **4.** 42. **5.** 3, 3, 3, 5, 5, 5; 15, 15, 15.

EXERCISE

93.— **6.** 16, 17, 18. **7.** (a) 225; (b) 29; (c) 1; (d) 65; (e) 144; (f) 18; (g) 385. **8.** A, \$750; B, \$1000; C, \$1750. **9.** 22 yd., 18 times. **10.** (a) 2808; (b) 660; (c) 42840; (d) 20160; (e) 1512; (f) 12493, 8303; (g) 1668167. **11.** 39312. **12.** 187. **13.** $3\frac{3}{4}$. **14.** (a) 90, (b) 21, (c) 5, (d) 4. **15.** $2\frac{2}{3}$. **16.** (a) $2\frac{1}{15}$, (b) 9, (c) $18\frac{67}{150}$, (d) $5\frac{3}{5}$, (e) 11, (f) $\frac{8}{84}$, (g) $3\frac{10}{33}$. **17.** (a) $\frac{48}{55}$, (b) $14\frac{2}{5}$, (c) $\frac{8}{32}$, (d) $1\frac{4}{5}$. **18.** (a) $\frac{8}{7}$, (b) $\frac{8}{25}$, (c) $\frac{5}{6}$, (d) $\frac{11}{7}$, (e) $\frac{6}{55}$, (f) $\frac{125}{242}$. **19.** $\frac{1}{11}$. **20.** (a) $\frac{1}{2}$, (b) $\frac{5}{24}$, (c) $\frac{11}{20}$, (d) $\frac{2}{3}$, (e) $\frac{1}{8}$. **21.** (a) $2\frac{2}{3}$, (b) \$3·50, (c) 15 yd. **22.** $\frac{1}{4}$, \$320. **23.** $\frac{10}{9}$, $133\frac{1}{3}$ ac. **24.** $\frac{16}{9}$, \$3·20. **25.** \$250, \$375. **26.** \$9000. **27.** $10\frac{1}{2}$ bu. **28.** $\$9\frac{5}{8}$. **29.** \$25200. **30.** $1\frac{209}{240}$ in. **31.** $\frac{1}{6}$. **32.** $1\frac{1}{5}$ lb. cornmeal, $\frac{3}{5}$ lb. flour, $\frac{3}{10}$ oz. sugar, $1\frac{2}{5}$ oz. lard, $1\frac{3}{5}$ oz. baking-powder. Cost $13\frac{41}{50}$c. **33.** \$900.

94.— **1.** (a) 446·89, (b) 583·0971. **2.** (a) 15·222, (b) 192·8175, (c) 20·40789. **3.** (a) 349·9, (b) 98·6303, (c) 38·1083. **4.** (a) 5·19, (b) ·3726099, (c) 322·873, (d) 71·84136.

95.— **1.** (a) 2·5, (b) 1·72, (c) ·0033, (d) ·134, (e) ·00023, (f) ·1426, (g) ·00258, (h) 3·4567, (i) ·00000542, (j) 3·253. **2.** (a) 1370, (b) 460000, (c) 10·14, (d) ·0377, (e) ·708, (f) 1·25, (g) ·3606, (h) ·5472. **3.** 43·725, 4·3725, ·43725, ·043725. **4.** 43·68, ·00375. **5.** ·8, ·05, ·016, ·875, ·0625, ·08, ·45. **6.** (a) 6·7246 +, (b) ·0365, (c) ·3529 +, (d) ·4285 +, (e) ·0769 +, (f) 7·3684 +. **7.** (a) ·0137, (b) ·67, (c) 111. **8.** (a) 6·724 +, (b) 17·660 +. **9.** (a) 10·8, (b) ·0000322, (c) 4·5, (d) 10. **10.** ·32, 32, 3·2. **11.** 13500 lb. **12.** 327 bu. **13.** 38·75 mi. **14.** 36·25, \$494·81 +. **15.** \$198·24 +. **16.** \$·84 +. **17.** 673·10 + gal. **18.** 28·34 +. **19.** ·679 +. **20.** 7·4 in. **21.** \$7·20.

EXERCISE

96.— 1. (a) $·023727, (b) ———.   2. (a) $200·62, (b) $224·76, (c) $272·41, (d) $321·77, (e) $661·80, (f) $812·98.   3. (a) $·019833, (b) ———, (c) (i) $167·11, (ii) $227·42, (iii) $267·06. 4. (a) $·182075, (b) $288·93, $644·89, $1246·66, $226·12, $999·94.

97.— 1. $\frac{8}{10}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{87}{100}$, $\frac{9}{20}$, $\frac{16}{25}$.   2. $\frac{1}{20}$, $\frac{1}{40}$, $\frac{1}{1000}$, $\frac{27}{40}$, $\frac{817}{1000}$, $\frac{18}{125}$.   3. $\frac{59}{80}$, $\frac{7}{625}$, $\frac{7}{400}$, $\frac{387}{5000}$, $\frac{129}{400}$.   4. $4\frac{3}{4}$, $10\frac{1}{4}$, $12\frac{8}{125}$, $7\frac{11}{40}$, $15\frac{3}{25}$.

98.— 2. ·1875, ·4375, ·68, ·28125, ·234375, ·92, ·552, ·0112, ·425, ·4875, ·228.   3. ·6666....., ·83333....., ·6363....., ·6153....., ·4666....., ·8235....., ·6111...., ·6956....., ·6428....., ·5446.......   4. ·8574....... 5. 60c.   6. ·15625 of stock left, $1000.   7. $\frac{7}{8}$, ·875, $2·10.   8. ·609375, $1·95.   9. ·2, $·74. 10. ·625, $14·50.

99.— 1. 425 cu. ft.   2. 373·85 lb.   3. 6 hr. 41·88 min. or 401·88 min.   4. 3 in.   5. ·305....... 6. 16·88.......   7. 96·57...... lb.   8. Each man $8, each boy $1·12.   9. $20.   10. 7·38 lb. 11. 47·52 tons.   12. $3000.   13. 29·529 + mi. 14. ·008792.   15. 8·32.   16. ·42.   17. $137·678. 18. $37·83.   19. 48·83°.   20. 2·06 + in. 21. 258315·84 gal.

100.— 1. Balance $220·15. 2. $8·65. 3. Balance $20·90. 4. Balance $1005·00.   5. $20·61. 6. Balance $466·50.   7. Profit $42·71.   8. Balance $324·97. 10. $10 × 4, $5 × 15, $2 × 9, $1 × 15; coins 64c.   11. $34·00.

101.— 1. $\frac{7}{4}$, $\frac{7}{16}$, $\frac{3}{40}$, $\frac{24}{25}$, $\frac{1}{160}$, $\frac{1}{32}$, $\frac{89}{4000}$.   2. $73\frac{1}{3}$%, 1·6%, $7\frac{1}{2}$%, $87\frac{1}{2}$%, ·857% +, $3\frac{1}{3}$%, 50·56%.   3. $11·55, £8 8s., ·0785, ·105, 22·5.   4. 46, 47 ft. 8 in., $4, $1\frac{1}{2}$, $4\frac{4}{5}$.   5. $33\frac{1}{3}$%, 300%, $7\frac{1}{7}$%, $16\frac{2}{3}$%, $31\frac{1}{4}$%.

EXERCISE

101.— **6.** \$360. **7.** 2751 marks. **8.** 7500, 9000. **9.** 299.
10. $16\frac{2}{3}$ lb. **11.** \$4500. **12.** $\frac{3}{10}$, 30%.
13. 52·59% +. **14.** 16%. **15.** $98\frac{1}{2}$%.
16. 13 yd. 6 in. + . **17.** 28%. **18.** (a) Toronto
·622 +, Buffalo ·603 +; (b) Toronto ·629 +,
Buffalo ·597 + . **19.** Sulphate of ammonia
320 lb., acid phosphate 1142·8 lb., muriate of
potash 160 lb., sand 377·2 lb. **20.** $44\frac{4}{9}$%,
$58\frac{1}{3}$%, 25%. **21.** 4·69% +. **22.** (a) 98, 196,
245, 294, 392, 441, 490, 588, 735; (b) 1250.

102.— **1.** $\frac{1}{6}$, $16\frac{2}{3}$%. **2.** 120%. **3.** $10\frac{2}{9}$%. **4.** \$120,
\$160, $\frac{2}{8}$, $\frac{4}{3}$. **5.** \$22,500. **6.** 15%. **7.** 25%.
8. \$48. **9.** \$400. **10.** \$1669·85. **11.** \$9·66.
12. $37\frac{1}{2}$%. **13.** 96c. **14.** $33\frac{1}{3}$%, 25%. **15.** 35%.

103.— **1.** \$37·50. **2.** \$18·75. **3.** \$1741·50. **4.** \$219·45.
5. \$4·32. **6.** $2\frac{1}{2}$%. **7.** $1\frac{1}{2}$%. **8.** $1\frac{1}{4}$%. **9.** \$1140,
\$1100. **10.** \$930, \$954·80. **11.** 150. **12.** \$160.
13. \$31·50, 90c. **14.** \$118, \$282. **15.** \$3.
16. 30,000 yd., \$90, $\frac{3}{20}$. **17.** \$60,000.
18. 29,400 lb., \$60. **19.** \$17·28. **20.** \$1680.
21. \$1962·52.

104.— **1.** \$1·25, \$11·25; \$1·22, \$14·03; \$5·15, \$20·60;
\$7·50, \$42·50; \$67·50, \$157·50; \$156·50, \$469·50;
\$1,593·75, \$2,656·25; \$25·49, \$611·76; \$327·60,
\$655·20; \$97·94, \$3,166·86. **2.** \$450, \$1,866·75,
\$2,400, \$922·50, \$1,744·20. **3.** \$478·80, \$422·14,
\$2,650·50, \$2,478·60, \$523·26. **5.** 10%. **6.** 35%.
7. \$250. **8.** \$20. **9.** \$42. **10.** \$4. **11.** \$2·88.
12. \$4·50. **13.** \$660. **14.** \$12·60. **15.** \$2·21.
16. \$7·95. **17.** \$42. **18.** \$31·50. **19.** \$78·92
+. **20.** \$70·19. **21.** \$60·04.

EXERCISE

105.— **2.** 5½, 8¼, 12½, 18. **3.** \$15, \$24·92, \$60, \$176,444·36, \$3170·72. **4.** \$410,000, \$150,000, \$2,750,000, \$9,125,000, \$4,850,000. **5.** \$18, \$1882. **6.** \$6. **7.** \$13. **8.** 80%. **9.** \$160,875. **10.** \$76,882·81. **11.** \$6,606. **13.** \$30·58, \$3·62. **15.** \$22·80, \$21·96, \$80·23.

106.— **1.** (a) \$22·50, (b) \$3·125, (c) \$112·50, (d) \$231, (e) \$12·60, (f) \$36·70. **2.** \$54, \$234. **3.** \$40. **4.** \$160. **5.** \$457·80. **6.** (a) \$271·62½, (b) \$1744·71 +. **7.** \$35, 25%. **8.** 16⅔%. **9.** (a) \$706·50, (b) \$2287·80, (c) \$847·80.

107.— **1.** \$3·75. **2.** \$14·30. **3.** \$180. **4.** \$2560. **5.** \$3200. **6.** $\frac{3}{20}$%. **7.** 2⅔%. **8.** \$1905. **9.** \$1125. **10.** \$33·20. **11.** \$4·60. **12.** \$4955. **13.** \$2500. **14.** \$4998. **15.** \$3240, \$2160, \$3600. **16.** \$10,000. **17.** 1$\frac{1}{10}$%.

108.— **1.** \$105·60. **2.** \$12, \$28, \$3·75, \$12·25, \$22·65. **3.** \$57·81, \$486·06. **4.** \$90·06 int. \$652·91 amt., \$65·24 int. \$4265·24 amt., \$118·59 int. \$3843·59 amt., \$289·04 int. \$6539·04 amt., \$242·60 int. \$10242·60 amt. **5.** \$300. **6.** \$500, \$2400, \$1200, \$14600. **7.** 5 yr. **8.** 6 mo., 3$\frac{63}{107}$ yr. **10.** 4%, 6%, 3½%. **11.** 4½%, 5%, 3%. **12.** \$252·77. **13.** Oct. 6th, 1918, \$127·51. **14.** \$1·83. **15.** \$301·99. **16.** \$507·60. **17.** \$138·22. **18.** \$201·10. **19.** (a) \$·63, \$·73; (b) \$1·59, \$1·86; (c) \$·85, \$·99.

109.— **1.** April 8th, 1919, \$295·41. **2.** (a) Dec. 6th, 1918, 63 da., \$3·45, \$396·55; (b) Aug. 4th, 1919, 93 da., \$14·01, \$985·99; (c) Oct. 11th, 1919, 99 da., \$81·37, \$5918·63; (d) Oct. 7th, 1918, 73 da., \$66, \$5934; (e) Dec. 13th, 1919, 146 da., \$36, \$1464.

109.— **3.** $\frac{441}{36600}$, $\frac{36059}{36600}$, \$3650.   **4.** \$1460.   **5.** \$3650.
6. Aug. 9th, 1920, \$615·21. **7.** \$608·26. **8.** (a) Aug.
18th, 1919, \$2531·85, 76 da., \$36·90, \$2494·95;
(b) Dec. 17th, 1920, \$1220·55.

110.— **1.** Gain \$50·00.   **2.** \$365·40, \$157·50, \$812·00,
\$265·00.   **3.** ·771 or 77·14% +.   **4.** \$680.
5. $33\frac{1}{3}$%, $44\frac{4}{9}$%, $22\frac{2}{9}$%,   \$800·00,   \$1066·67,
\$533·33.   **6.** \$2,926.   **7.** $29\frac{3}{5}$%.   **8.** \$1180,
\$1150·50.   **9.** \$1·41$\frac{1}{3}$.   **10.** \$24·00.
11. \$320·50.   **12.** \$16·10.   **13.** \$590·83.
14. \$2·40. **15.** \$278·78. **16.** 8800 bu. **17.** \$55·25.
18. \$99·78. **19.** \$297·29. **20.** $52\frac{8}{21}$%. **21.** 24c.
22. \$4·50.   **23.** 90 lb.   **24.** $144\frac{7}{22}$%, $93\frac{81}{88}$%,
$10\frac{30}{47}$%, $14\frac{88}{133}$%.   **25.** \$38·25.   **26.** \$150.
27. 432 lb. nitrogen, 162 lb. phosphorus, 108 lb.
potash.   **28.** \$2·97.   **29.** \$12·32.   **30.** 4900 lb.

111.— **1.** 1089 ft.   **2.** $37\frac{1}{8}$ ft. × $24\frac{3}{4}$ ft.   **3.** Posts \$110,
wire \$122·50.   **4.** \$3990.   **5.** 648 sq. ft.   **6.** 800 ac.
7. $94\frac{1}{2}$ sq. ft.   **8.** \$19·50.   **9.** \$82·01.   **10.** 15
bundles.   **11.** $41\frac{1}{3}$ yd.   **12.** \$162.   **13.** $1\frac{3}{5}$ ac.
14. \$9900.   **15.** 51 min.   **16.** 133 yd. 1 ft. 6 in.
17. 20 ft. 10 in.   **18.** 144 cu. yd.   **19.** $253\frac{1}{3}$ cu. ft.
20. 11 ft.   **21.** 8 ft.   **22.** 705 lb. 15 oz.   **23.** 10·69
+ ft.   **24.** 268·758 + bu.   **25.** 18 tons.   **26.** 26
flues.   **27.** 2280960 bricks.   **28.** 62 sq. yd.   **29.** 864
blocks.   **30.** \$1260.   **31.** (a) $3678\frac{2}{3}$ bd. ft.;
(b) \$165·54.   **32.** $74\frac{4}{5}$ bd. ft. **33.** \$17128·50.

112.— **1.** (a) 334190,   (b) 26·43 +.   **2.** 1944 mi.
3. 14989 +.   **4.** \$3·00.   **5.** (a) \$297 loss,
(b) 132 da.   **6.** \$127·60 gain.   **7.** (a) Quotient
622, Remainder 7; (b) H.C.F. 9, L.C.M. 568008.
8. (a) $1\frac{49}{60}$, (b) $15\frac{23}{88}$.   **9.** (a) $18\frac{9}{10}$; (b) 80.

EXERCISE

112.— 10. (a) ·1875, ·1360, ·1538, ·1739, ·1612; (b) $\frac{109}{128}$, ·848. **11.** The former. **12.** $24·92$\frac{4}{13}$, $29·07$\frac{9}{13}$. **13.** 36 tucks. **14.** $48·48$\frac{1}{4}$. **15.** Balance $19·85. **16.** 96%, 90%, $83\frac{1}{3}$%, 80%, $87\frac{1}{2}$%. **17.** $2295. **18.** $820. **19.** 16 mills, $19·20. **20.** $327·04. **21.** $1523. **22.** $18·00. **23.** $3·00. **24.** $300, 5%. **25.** $371·70. **26.** $251·11. **27.** $60·32. **28.** $112\frac{2}{3}$ bd. ft. **29.** 348·48 tons. **30.** 3·072 in.

SECTION VI

113.— 1. 2^5, 3, 7, 11; 2^4, 3, 5, 103. **2.** (a) 213; (b) 2, (c) 23. **3.** (a) 7560, (b) 4464, (c) 1478235. **4.** (a) $10\frac{1}{2}$, (b) 48. **5.** (a) $\frac{59}{60}$, (b) $23\frac{59}{117}$, (c) $\frac{17}{55}$, (d) 28. **6.** (a) $\frac{9}{14}$, (b) $\frac{2}{27}$. **7.** (a) 27·0410356, (b) ·0046, (c) 13·14. **8.** (a) $\frac{279}{154}$, (b) 160 ft. 4 in. +. **9.** (a) 543 lb., (b) $12·41$\frac{1}{2}$. **10.** $287, $137. **11.** $502·12$\frac{1}{2}$. **12.** $460. **13.** (a) 1,457,238·08 lb., (b) 90,285·92 lb., (c) 20·88 lb. +, (d) $37,680·77, (e) $8,017·24, (f) $29,663·53, (g) $·019168 +. **14.** ·002746 +. **15.** $175, $325, $100. **16.** 540 ft. **17.** $119·45. **18.** 10 times. **19.** $327\frac{3}{5}$ lb., 306 lb. **20.** 2460 loaves. **21.** 36%. **22.** $19·80. **23.** $108. **24.** $185·29. **25.** $9600. **26.** $98·78. **27.** $500. **28.** $100,000.

114.— 1. $562·43, $62·43. **2.** $220·50, $20·50; $3787·43, $787·43; $2331·83, $331·83; $5508·11, $708·11. **3.** $324·73, $24·73. **4.** $842·74, $42·74; $2081·21, $81·21. **5.** (a) $790·08, (b) $5964·58. **6.** $3528·68, $4564·66. **7.** $1226·10. **8.** (a) $2·45, (b) $10,377·70.

115.— 2. $43·69, $4·37, 4c., $15·19, $2·76. **3.** $3·11, $2·69, $1·65, $49·50. **4.** $145·15, $127·80, $46·39, $2·75. **5.** $165·34, $45·75, $4·53, 5c.

EXERCISE

115.— **6.** 2c., 27c., 95c., 20c.   **7.** $1·06, $177·21, $3031·14, $1077·47.

116.— **1.** $309·07, $9·07; $418·27, $18·27; $807·96, $57·96.   **2.** $679·17, $39·17; $1324·90, $124·90; $460·03, $35·03.   **3.** $861·51, $61·51; $275·95, $25·95; $141·43, $16·43.   **4.** $249·31, $14·31; $556·45, $76·45; $530·68, $30·68.   **5.** $1352·91; $1725·22; $3226·98.   **6.** $21·38.

117.— **1.** $2500, $6025; $6000, $4455; $5700, $3605·25; $2300, $621.   **2.** 60, 41, 55.   **3.** $216, $1150, $308.   **4.** 500, 3625.   **5.** $4·50, $8·25.   **6.** (a) $4\frac{1}{4}\%$, (b) $4\frac{1}{4}\%$, (c) $5\frac{1}{3}\%$.   **7.** 26.   **8.** $350.   **9.** $3400, $6·25.   **10.** $1233.   **11.** $640.   **12.** $480.   **13.** $100·80.   **14.** 180 shares, $18,000, $760.   **15.** 40 shares, $240, $\frac{1}{20}$, 5%.   **16.** $6\frac{2}{3}\%$.   **17.** 8%.   **18.** $9 larger.   **19.** $4600.   **20.** $9000, $7200.   **21.** $160.

118.— **1.** $4·13, $14·10, $26·60, $30·23, $47·90, $75·24, $93·60.   **2.** A 40c. note and 5c. stamp, total 46c.; a 90c. note, total 97c.; a $1·50 note, total $1·67; a $2·50 note and 40c. note, total $2·96; a $3 note and 70c. note, total $3·84; a $4 note and 80c. note, total $4·90; a $5 note and 90c. note, total $5·97; two $10 notes, a $5 note, and a $1 note, total $26·11.   **3.** 1c., 1c., 0, 1c., 1c., 3c.   **4.** $360·45, $2002·50, $236·80.   **5.** $1201·50.   **6.** $2406.   **7.** $159·30.   **8.** $3208.   **9.** $720.   **10.** $320.   **11.** $140.   **12.** $\frac{1}{3}\%$.   **13.** $6034·46, $675·50, $1144·80.   **14.** £1315·10, 24471·40 francs, 33542·97 marks.

119.— **2.** 144, 225, 324, 2025, 8100.   **3.** 3, 4, 7, 11, 40, 90.   **4.** ·5, $\frac{1}{6}$, ·7, 1·2, $\frac{8}{15}$.   **5.** 8 in., 3 yd., 60 rd., $\frac{1}{2}$ mi.   **6.** 32 in., 1·2 yd., 240 rd., 2 mi.

EXERCISE

119.— 7. 18, 33, 87, 99, 125, 256, ·51, 11·2, 726.
　　　8. 117 ft.　　**9.** 280 rd.

120.— 1. 62, 64, 73, 92.　　**2.** 141, 156, 193, 256, 313.
　　　3. 416, 527, 597, 809, 879.　　**4.** 1679, 2718, 6211.
　　　5. ·36, ·57, ·79, ·88, ·17, ·27.　　**6.** 2·7, 3·4,
　　　12·1, 7·01, 18·47.　　**7.** ·388, ·572, 34·91, 2·403.
　　　8. 1·414, 1·732, 2·236, 3·162, 4·123, 11·090, ·948,
　　　·714, 2·489, 2·061.　　**9.** $\frac{19}{24}$, $\frac{28}{29}$, $\frac{88}{49}$, $1\frac{3}{4}$, $5\frac{1}{12}$,
　　　$5\frac{17}{26}$.　　**10.** ·866, ·845, ·522, ·384, 2·121, 1·914.
　　　11. 5%.　　**12.** 33·541 rd.

SECTION VII

121.— 1. 120 sq. ft., 60 sq. ft.　　**2.** 121 sq. ft., 242 sq. ft.
　　　3. $47\frac{2}{3}$ sq. ft.　　**4.** $127\frac{1}{2}$ bd. ft.　　**6.** 24 ft.
　　　7. 15 in.

122.— 1. 25 sq. ft., 5 ft.　·　**2.** 15 ft.　　**3.** 25 ft.
　　　4. 31·24.......ft.　　**5.** 14·83.......ft.

123.— 1. 44 ft., $25\frac{1}{7}$ ft., $18\frac{6}{7}$ft., $62\frac{2}{7}$ in.　　**2.** $40\frac{6}{7}$ ft.
　　　$56\frac{4}{7}$ ft., $75\frac{6}{7}$ ft.　　**3.** $10\frac{1}{2}$ ft., $5\frac{8}{11}$ ft., $23\frac{12}{13}$ ft.
　　　4. 840 turns.　　**5.** 12 yd.　　**6.** 196 turns, $3\frac{4}{15}$ turns.

124.—1. 154 sq. ft., $86\frac{5}{8}$ sq. ft., $1,134\frac{4}{7}$ sq. yd., $1,886\frac{1}{2}$ sq.
　　　ft.　　**2.** $50\frac{2}{7}$ sq. ft., $176\frac{11}{14}$ sq. ft., $11,694\frac{4}{7}$ sq. in.,
　　　$113\frac{1}{7}$ sq. yd.　　**3.** $38\frac{1}{2}$ sq. ft., $86\frac{5}{8}$ sq. yd., $127\frac{8}{11}$
　　　sq. rd.　　**4.** $50\frac{10}{11}$ ac.　　**5.** 123·33....... yd.
　　　6. 154 sq. ft.; 1 ft. by 154 ft., 2 ft. by 77 ft., 7 ft.
　　　by 22 ft., 14 ft. by 11 ft.

125.— 1. 80 sq. ft., $85\frac{1}{4}$ sq. ft., $4\frac{1}{2}$ sq. ft.　　**2.** $293\frac{1}{3}$ sq. ft.
　　　3. $13·20.　**4.** $481\frac{1}{4}$ sq. yd.　**5.** 63 sq. in.　**6.** $10\frac{1}{2}$ ft.
　　　7. 176 sq. in.

126.— 1. 248 cu. ft., 248 sq. ft.　**2.** 600 cu. ft.　**3.** $3620\frac{4}{7}$
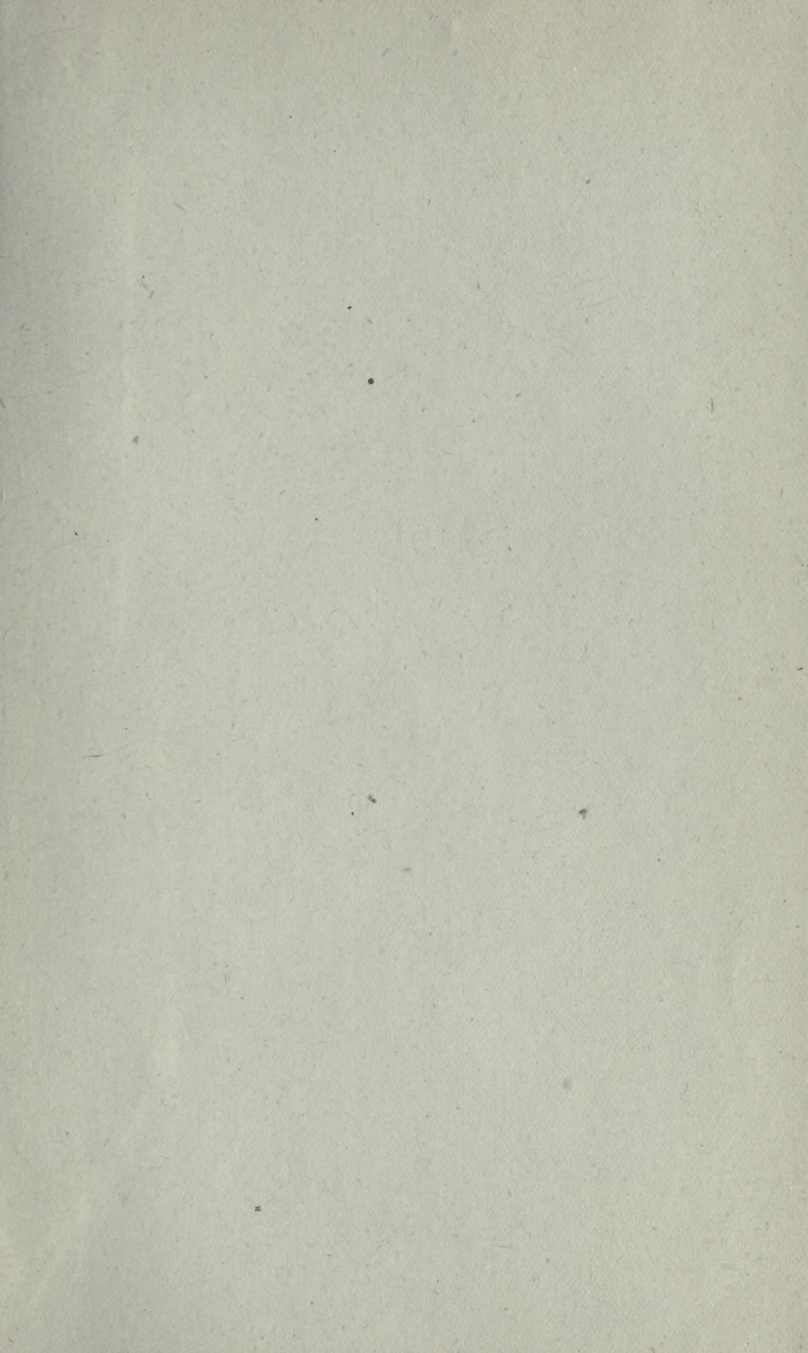　　　cu. ft.　**4.** 297 cu. ft., $1856\frac{1}{4}$ gal.　**5.** 7 in.

EXERCISE

127.— **1.** 80 ft.   **2.** $55687\frac{1}{2}$ bu.   **3.** 362 tons.   **4.** $2205\frac{34}{65}$
gal.   **5.** 18·176 rd.   **6.** 480 sq. in., 432 cu. in.
7. $63\frac{9}{14}$ cu. in.   **8.** 1628 sq. in., 4620 cu. in.
9. 80 ft. 10 in.   **11.** 13·8 ft., 36 ft.

SECTION IX

128.—1. $2152·37.   **2.** 56 lb., 60c.   **3.** (a) $2\frac{3}{4}$, (b) quo-
tient 12388, remainder 309.   **4.** $4444·80.   **5.** 6c.
6. 270 bu.   **7.** $542·90.   **8.** $171·25.   **9.** 18, 19,
20, 21.   **10.** 53.   **11.** 23,700.   **12.** 3 yd. 2 ft. 7 in.
13. $\frac{8}{9}$.   **14.** (a) $2\frac{7}{60}$, (b) $1\frac{3}{8}$, (c) $6\frac{15}{128}$, (d) $12\frac{3}{8}$.
15. 100 doz.   **16.** $4·76.   **17.** $149\frac{1}{3}$.   **18.** $3329·75.
19. $319·37\frac{1}{2}$.   **20.** A $32, B $80, C $20.   **21.** $129·09.
22. $30\frac{1}{4}$c.   **23.** $37·50.   **24.** 20 men.   **25.** $1068\frac{11}{24}$
lb., $2180·16\frac{1}{2}$.   **26.** $1119.   **27.** $4173·81 gain.
28. (a) 261·44, (b) 15, (c) ·70684.   **29.** ·752651 +.
30. 9·7875 yr.   **31.** $689·83.   **32.** $443·18\frac{2}{7}$.
33. $19·53 +.   **34.** 144,000 ac.   **35.** 4 hr., 48 min.
36. 2448 cu. ft., 4·195 ft.   **37.** 20 ft. and 6 ft.
38. $1512\frac{1}{2}$ lb.   **39.** $28·80.   **40.** 800 lb.   **41.** 25%.
42. $16\frac{2}{3}$% gain.   **43.** $16\frac{4}{11}$% gain.   **44.** $24\frac{16}{41}$%.
45. 80%, $468.   **46.** $1000 gain.   **47.** $325.
48. (a) —, (b) $310·62\frac{1}{2}$, (c) —.   **49.** 14 mills.
50. $4987·50, $12,987·50.   **51.** 6000 bu.   **52.** $54\frac{3}{4}$%.
53. $776·76.   **54.** $14·35.   **55.** $15·10.   **56.** $432·34.
57. $4\frac{1}{2}$%.   **58.** (a) Mar. 13th, 1920; (b) $1618·35,
(c) 69 da.; (d) $18·35; (e) $1600.   **59.** $4638·77.
60. $3\frac{3}{4}$%.   **61.** $40,800.   **62.** (a) $797·50;
(b) £250 15s. 8d. +.   **63.** 61·33 +, 261·25 +.
64. 50·596 rd., 44·848 rd.   **65.** 10 ft.